A Foundationally Verified Intermediate Verification Language

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How to verify functional correctness?

```
int minimum (int a[ ], int n) {
    int min = a[0];
    for(int i = 0; i < n; i++) {
        int j = a[i];
        if (j < min) min = j;
    }
    return min;
}
```

Dafny

```
method min (a: array<int>) returns (m: int)
requires a.Length > 0
ensures forall i : int :: 0 <= i < a.Length ==> m <= a[i]
ensures m in a[..]
{
    m := a[0];
    var i := 1;
    while (i < a.Length)
    invariant 1 <= i <= a.Length
    invariant forall j : int :: 0 <= j < i ==> m <= a[j]
    invariant m in a[0..i]
    {
        if (a[i] < m) {
            m := a[i];
        }
        i := i + 1;
    }
    return m;
}
```

Semi-Automated Verifiers

```
method min (a: array<int>) returns (m: int)
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    {
        if (a[i] < m) {
            m := a[i];
        }
        i := i + 1;
    }
    return m;
}
```

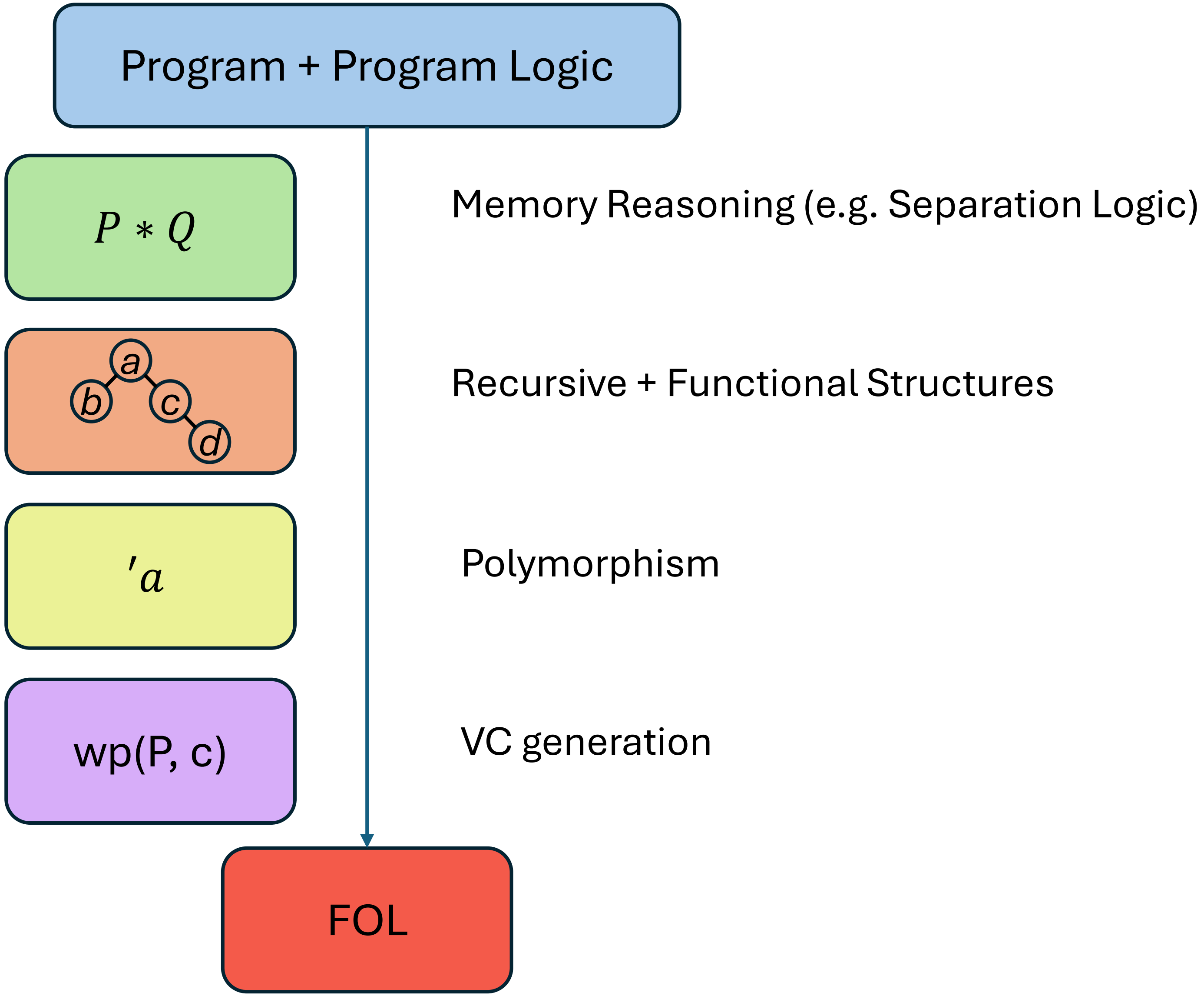
Verification Condition (VC) generator

$$\big((\mathrm{a[i]} < m) \rightarrow m' = a[i]\big) \wedge (\neg\,(a[i] < m) \rightarrow m' = m) \wedge \ldots$$

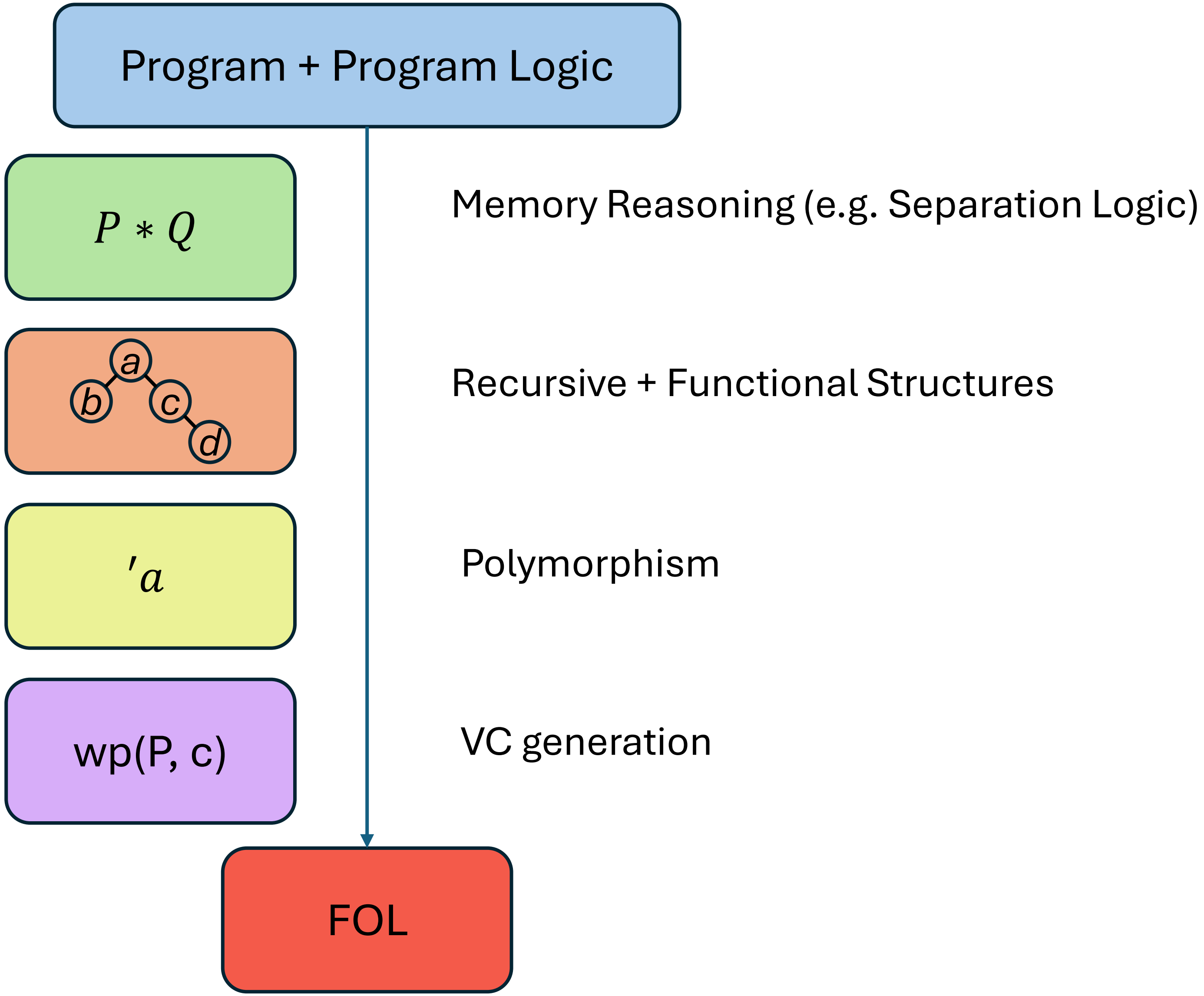
(SMT) Solver

Valid?

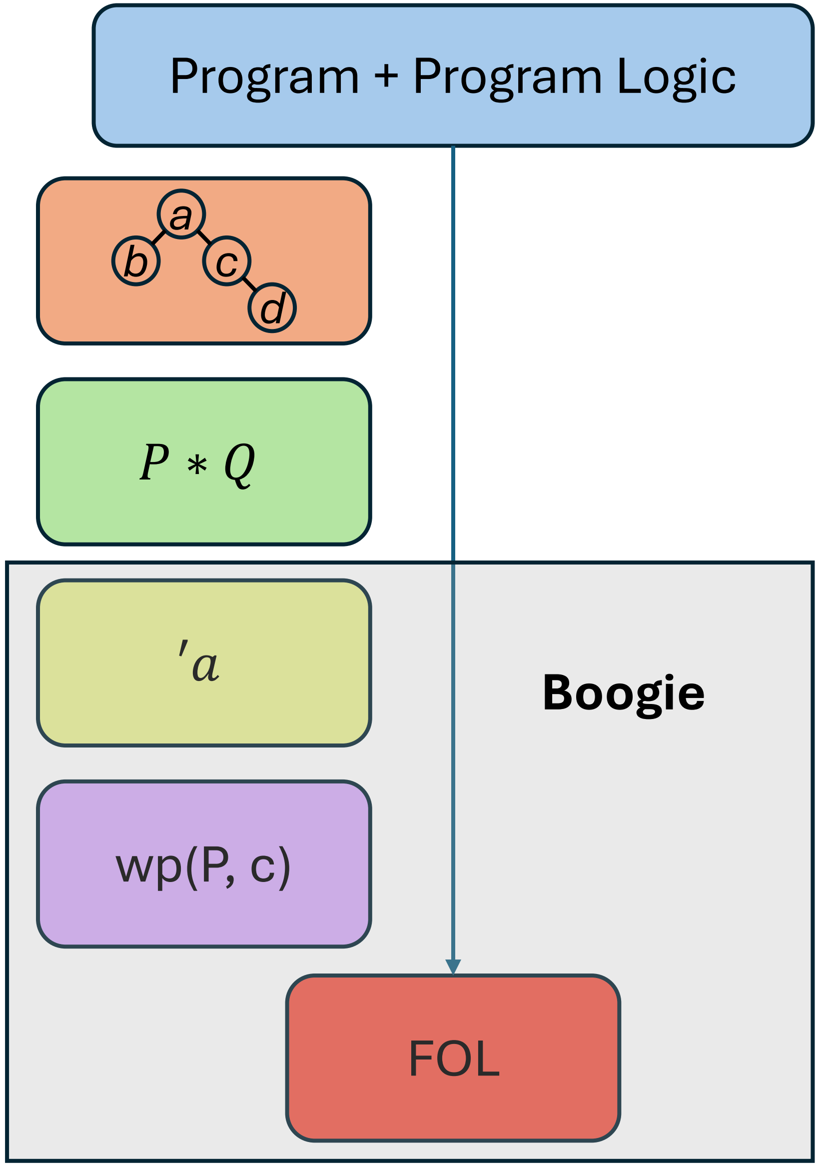
From Programs to FOL



From Programs to FOL



Intermediate Verification Languages (IVLs)



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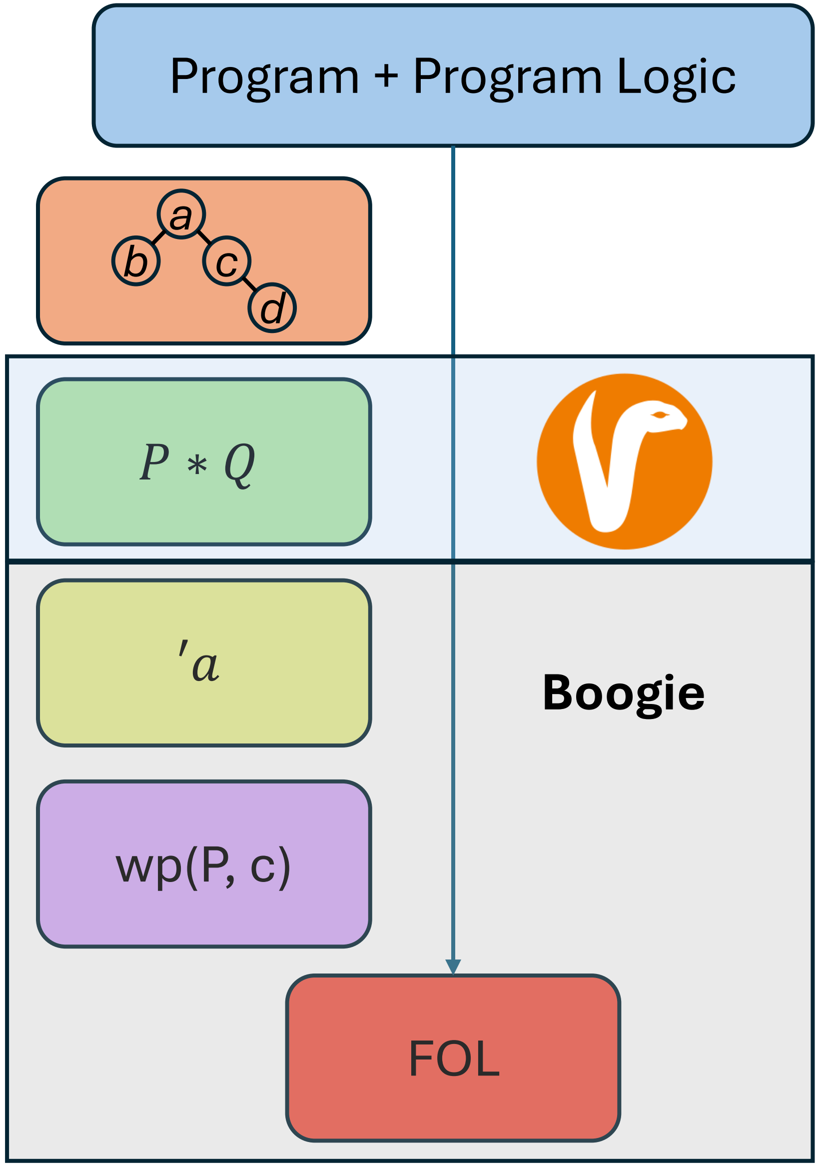

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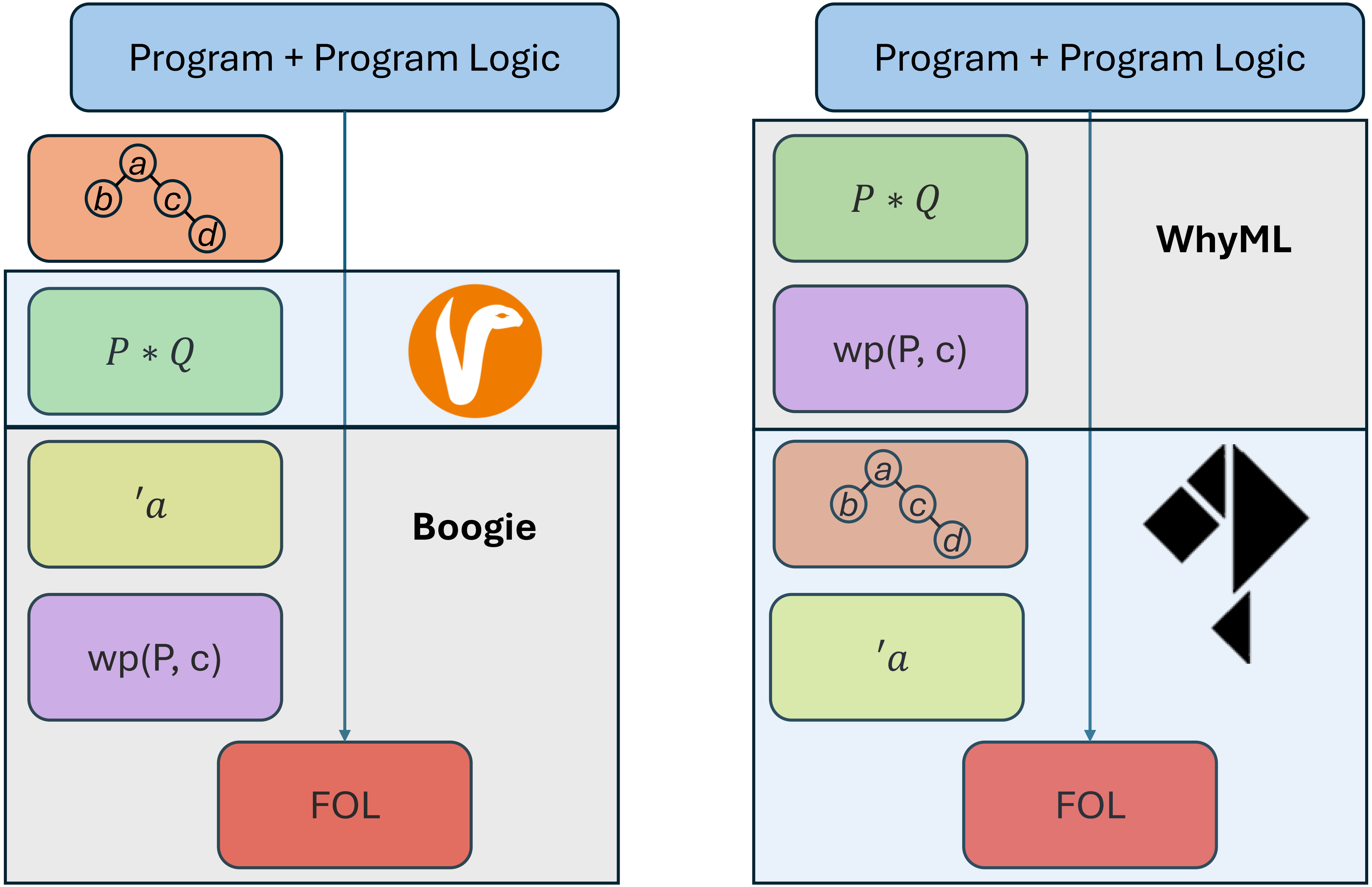

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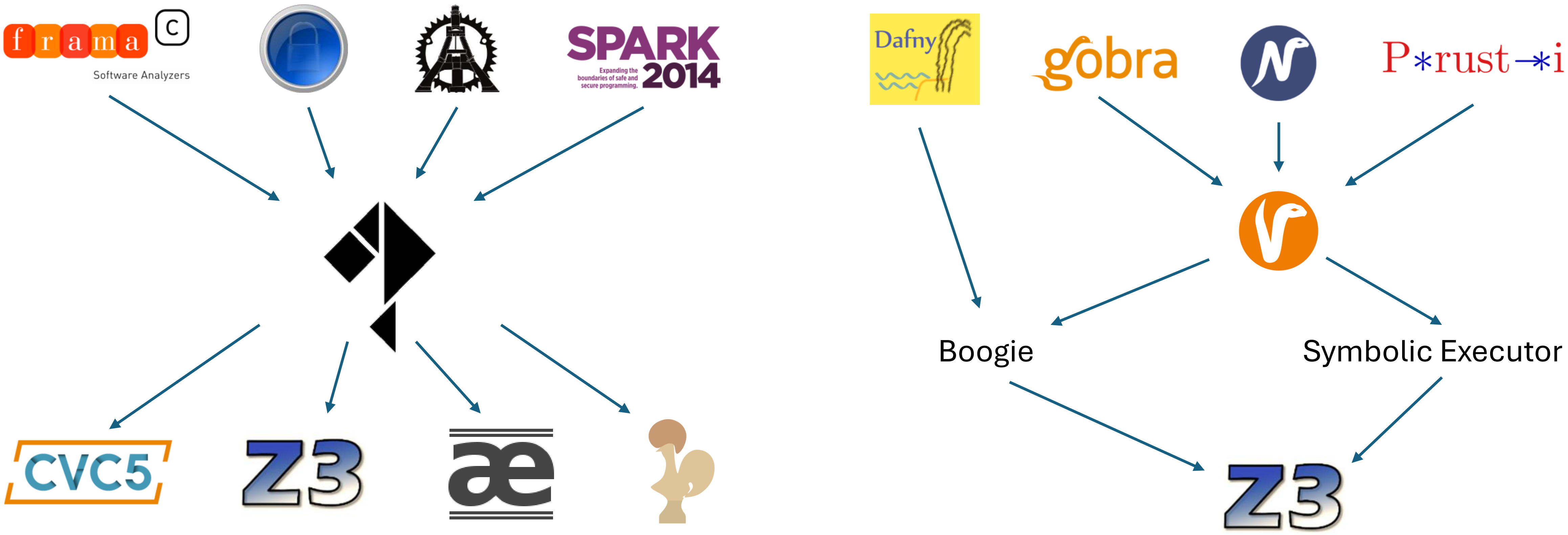

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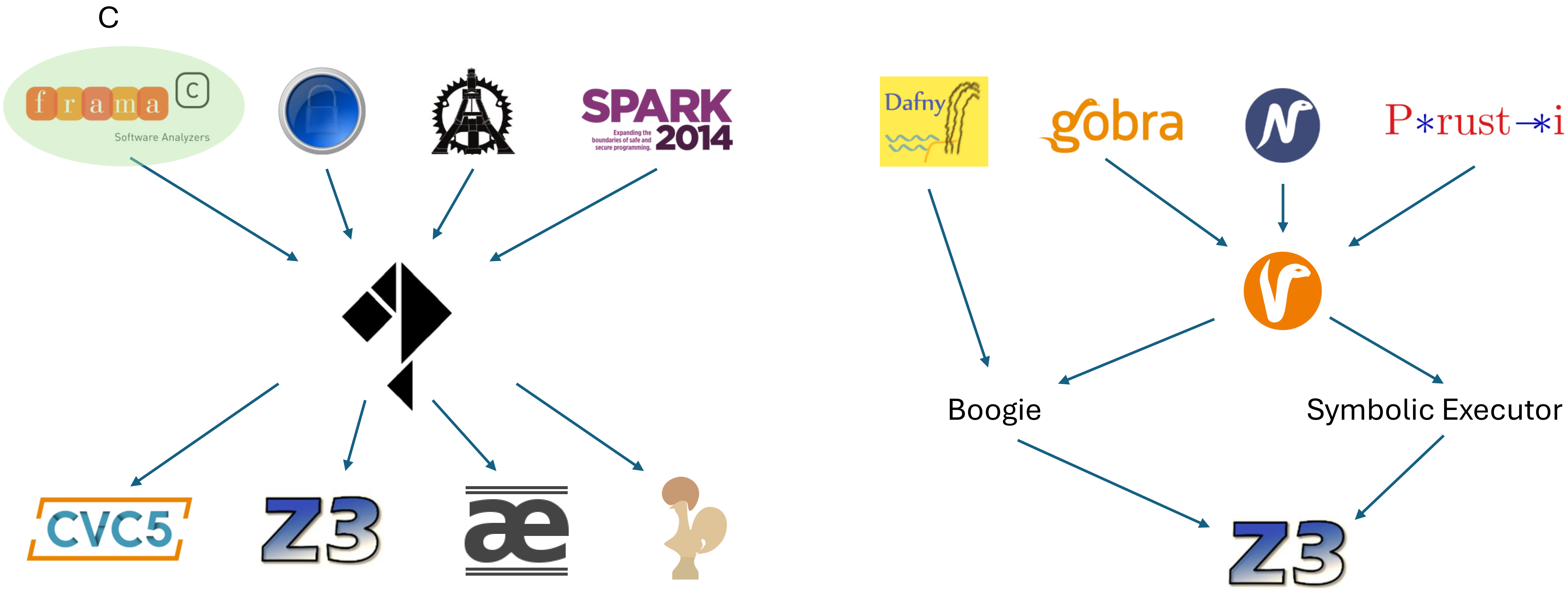

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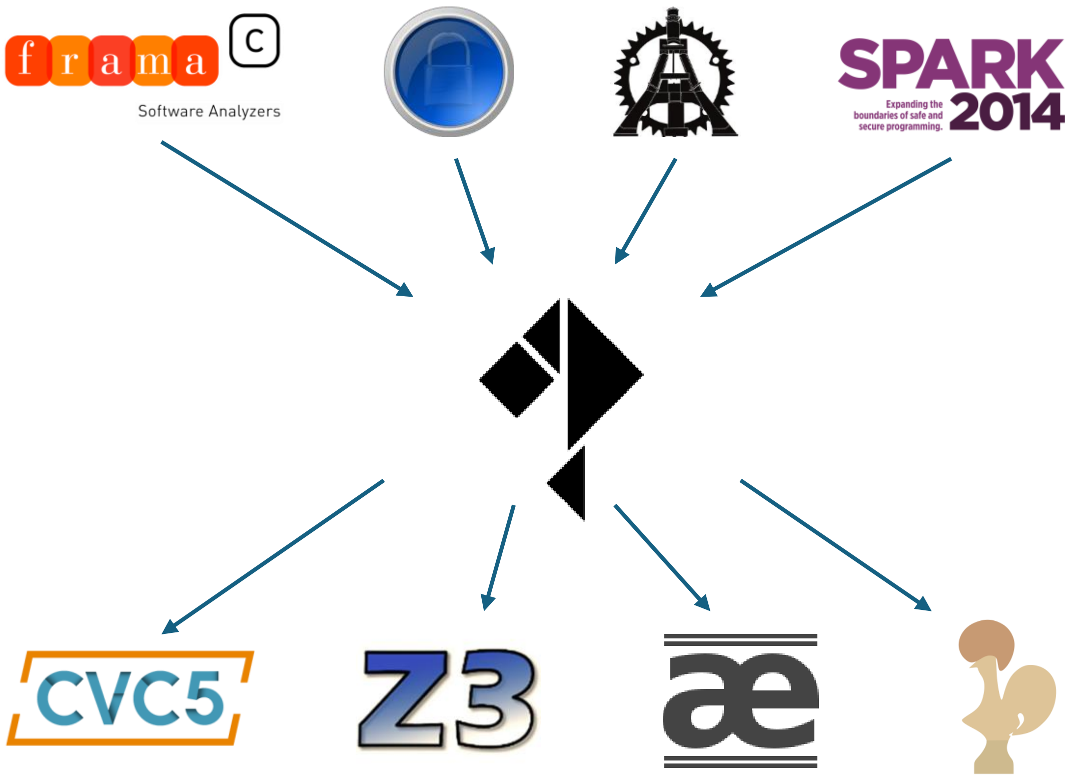

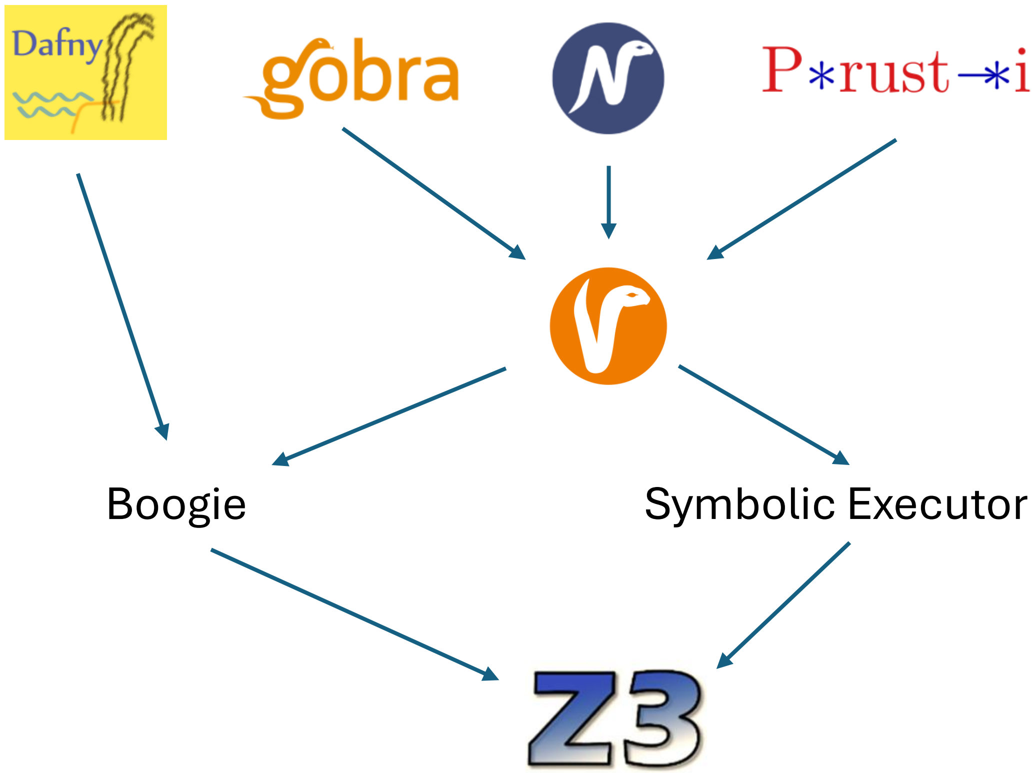

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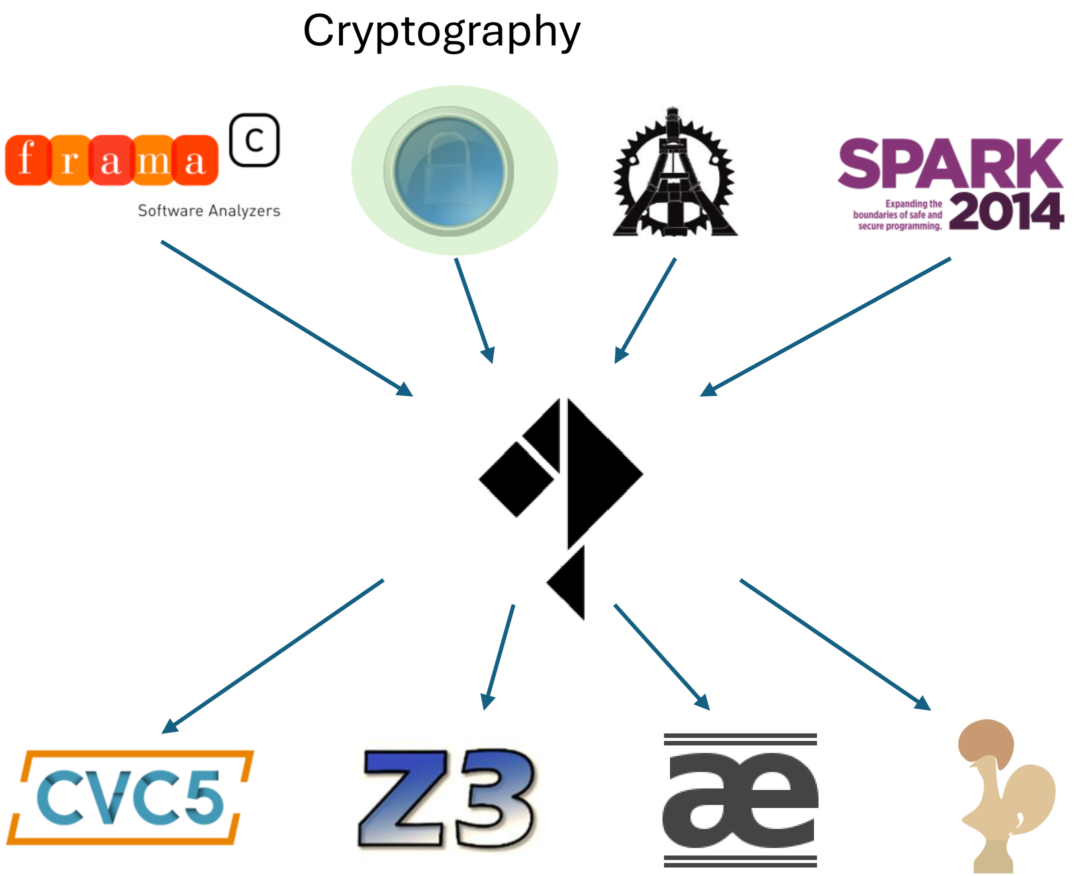

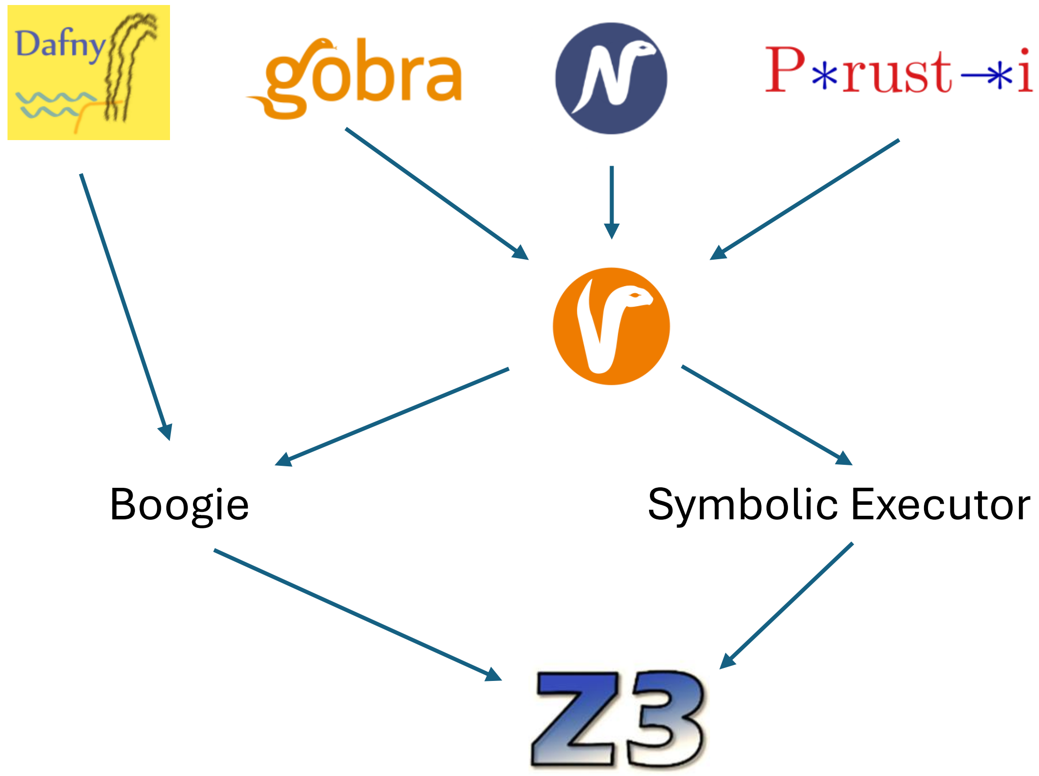

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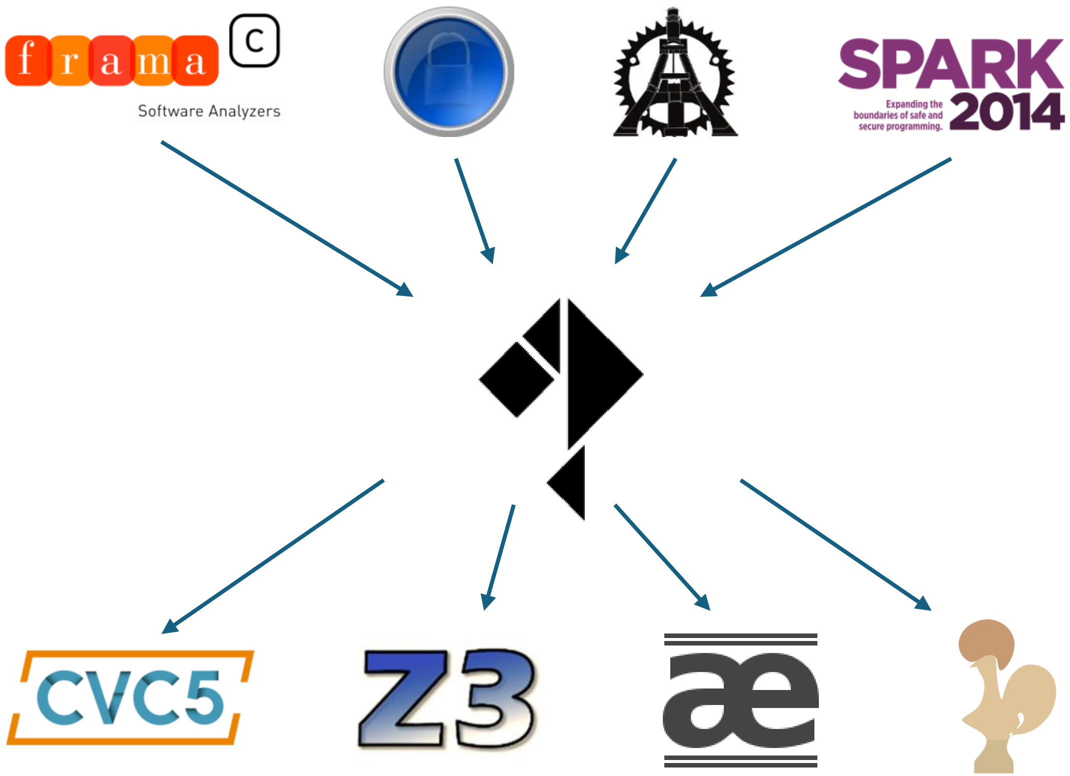

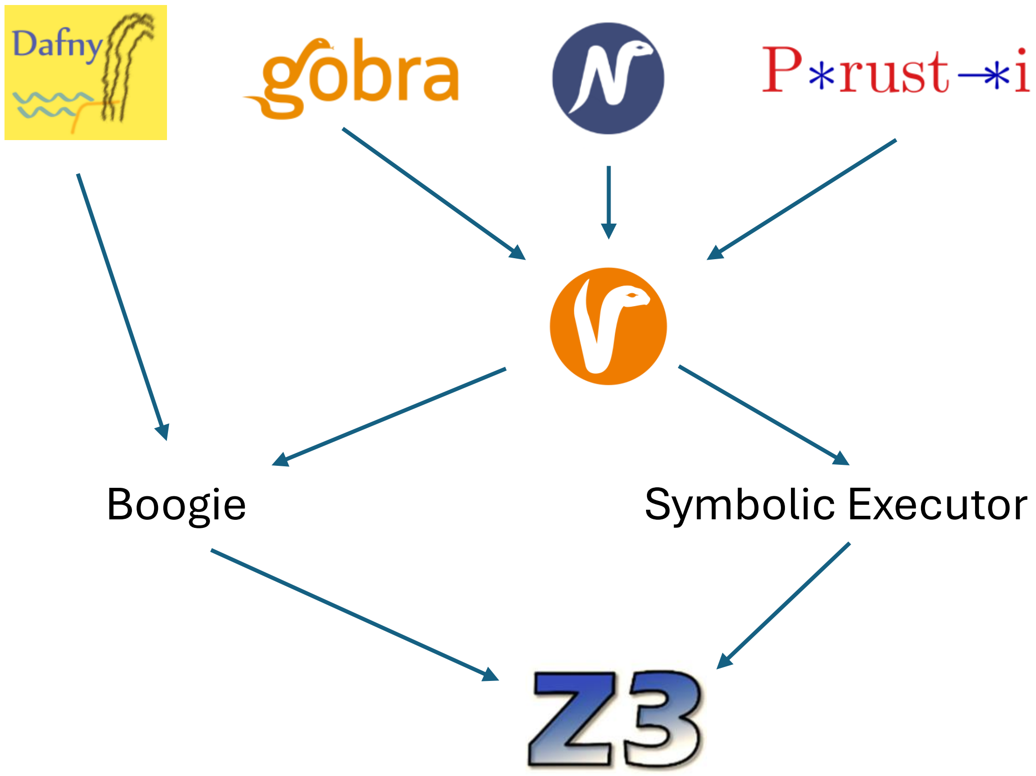

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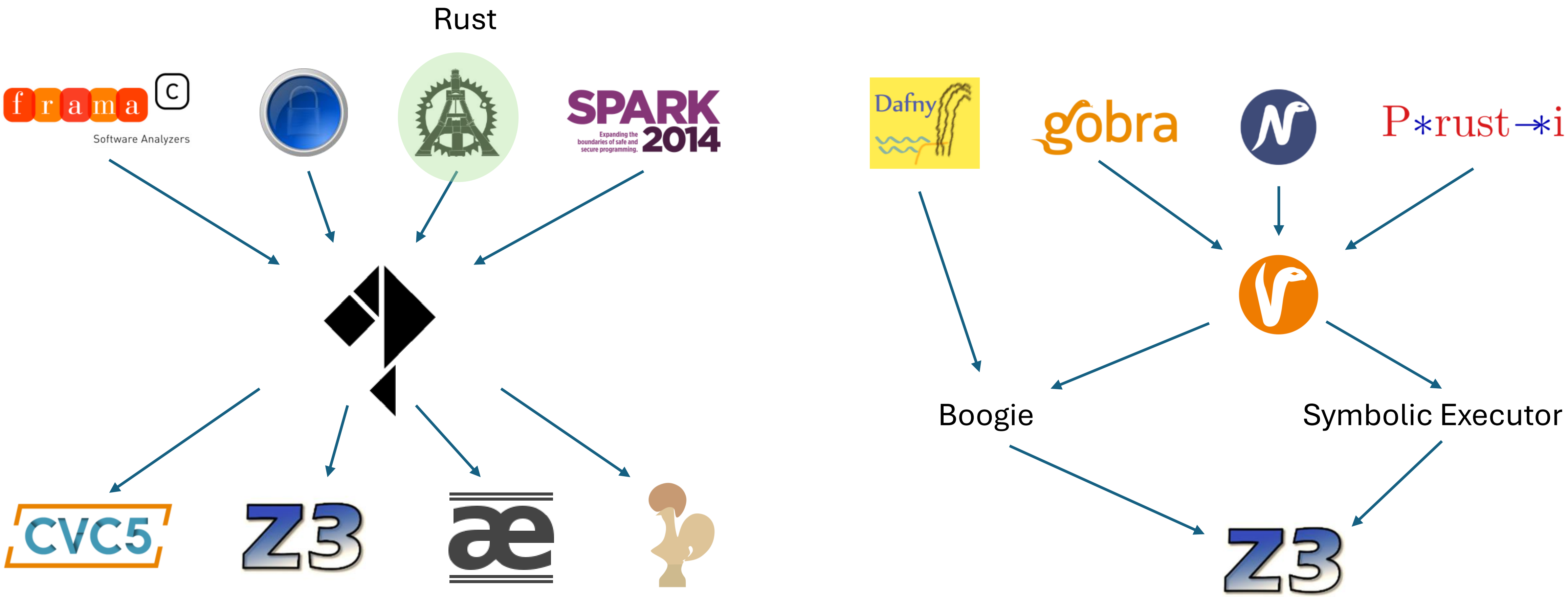

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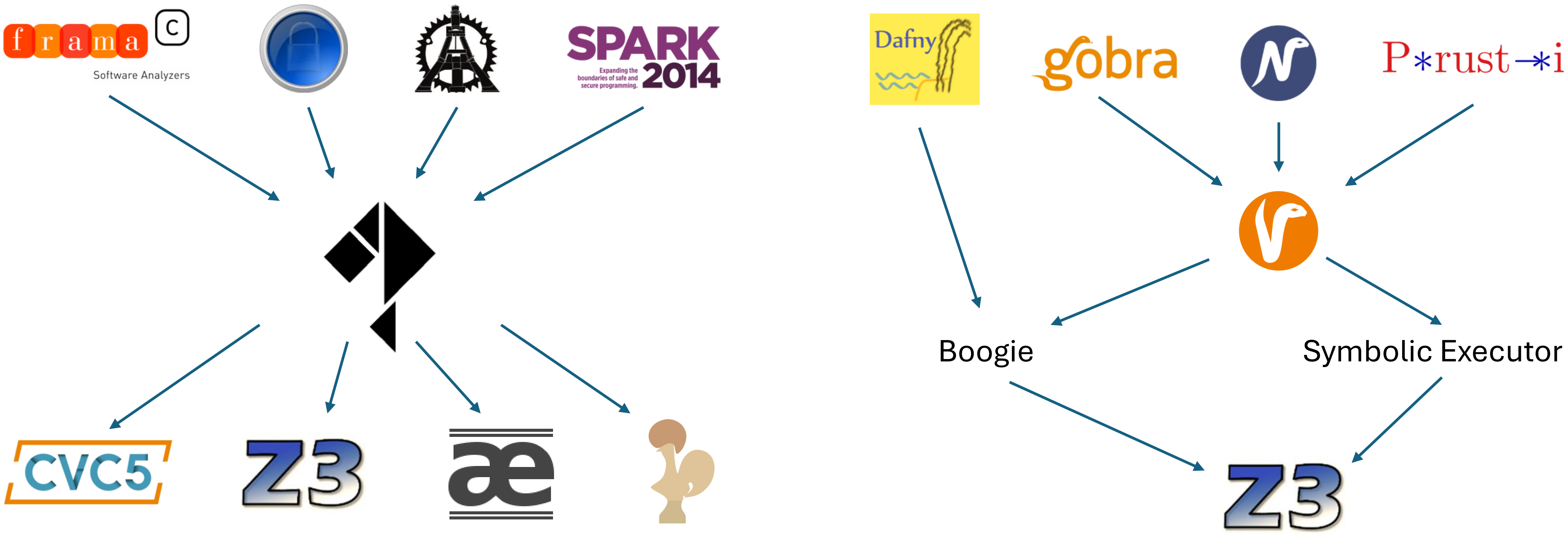

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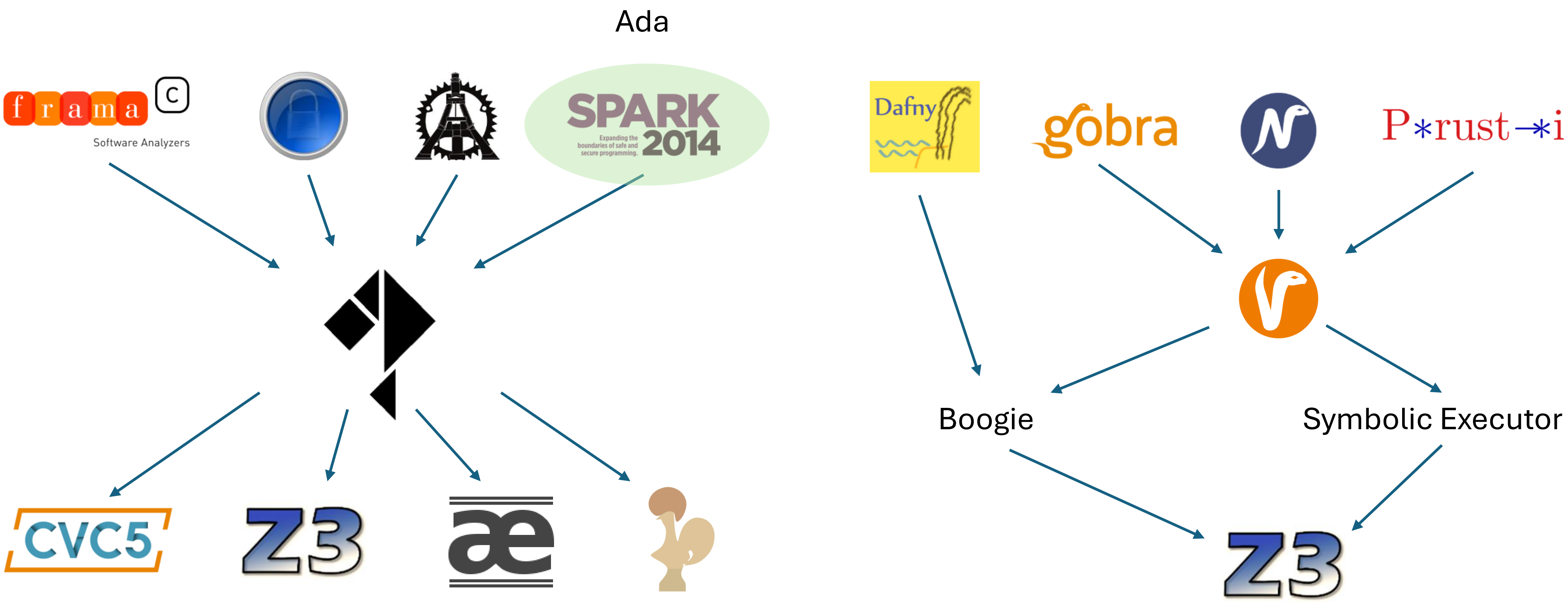

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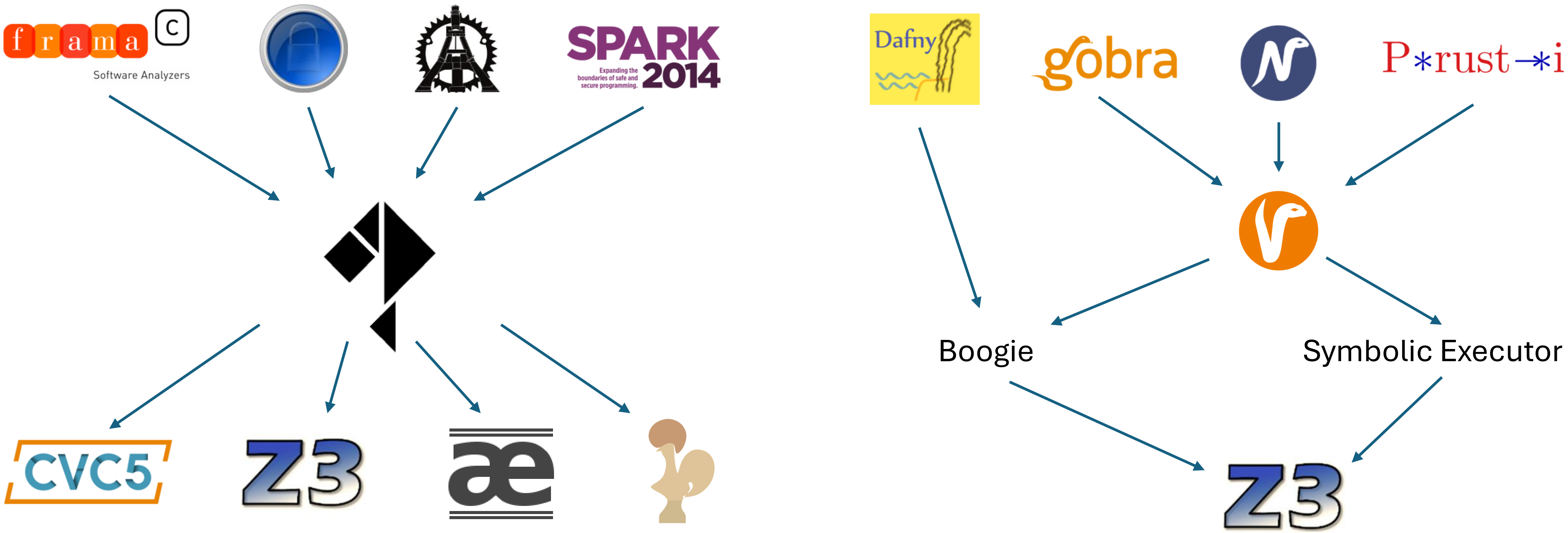

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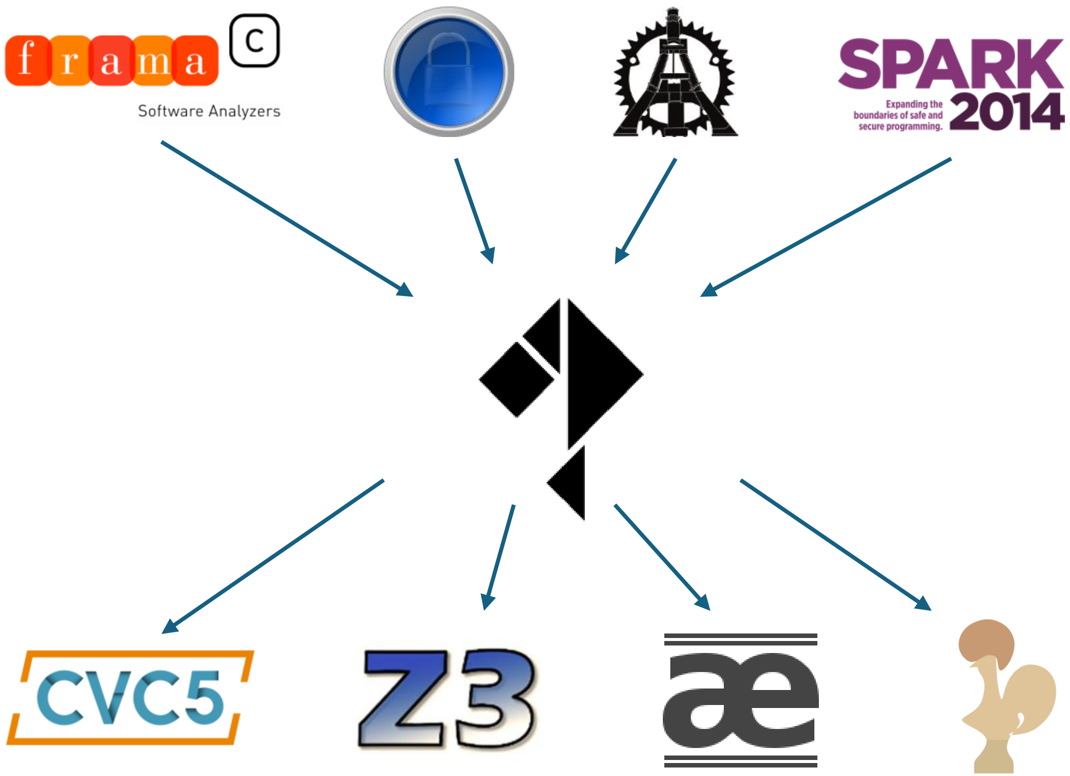

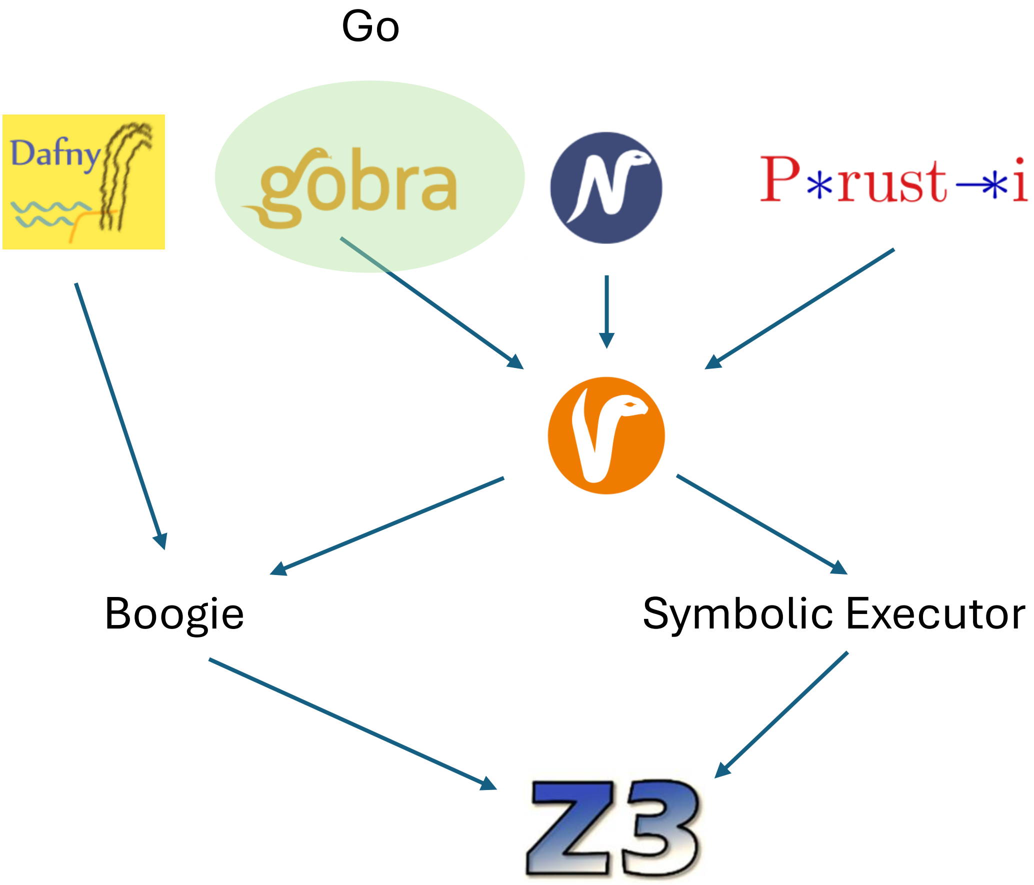

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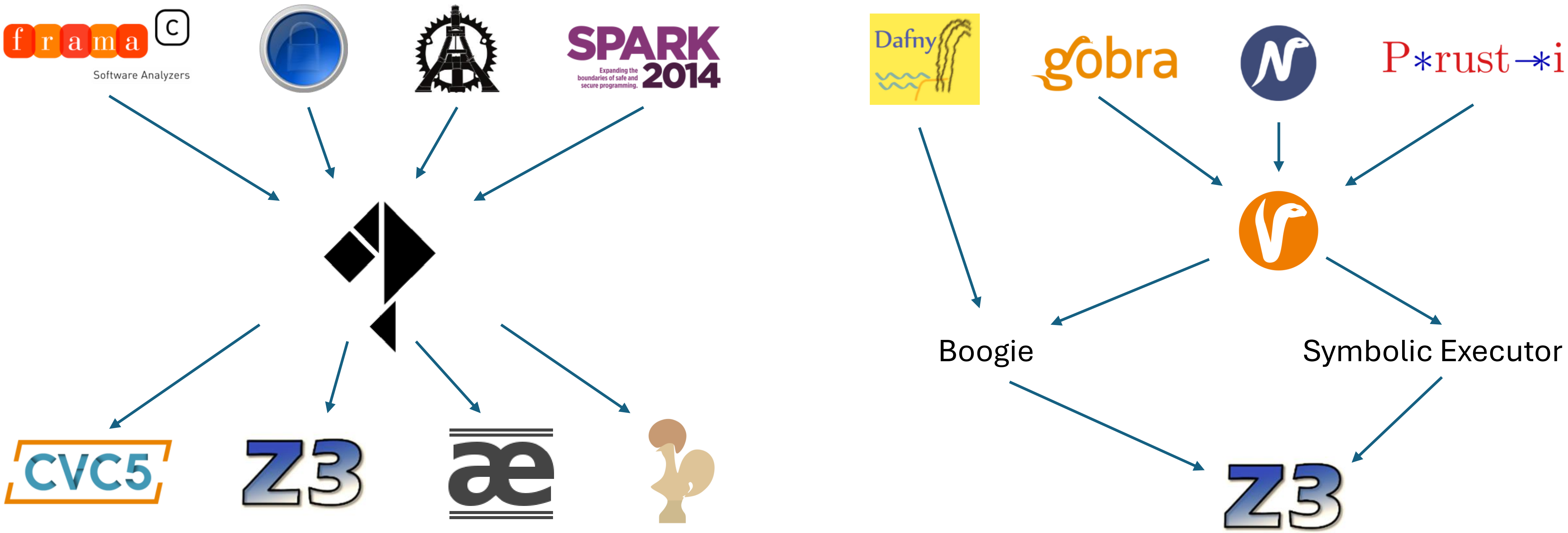

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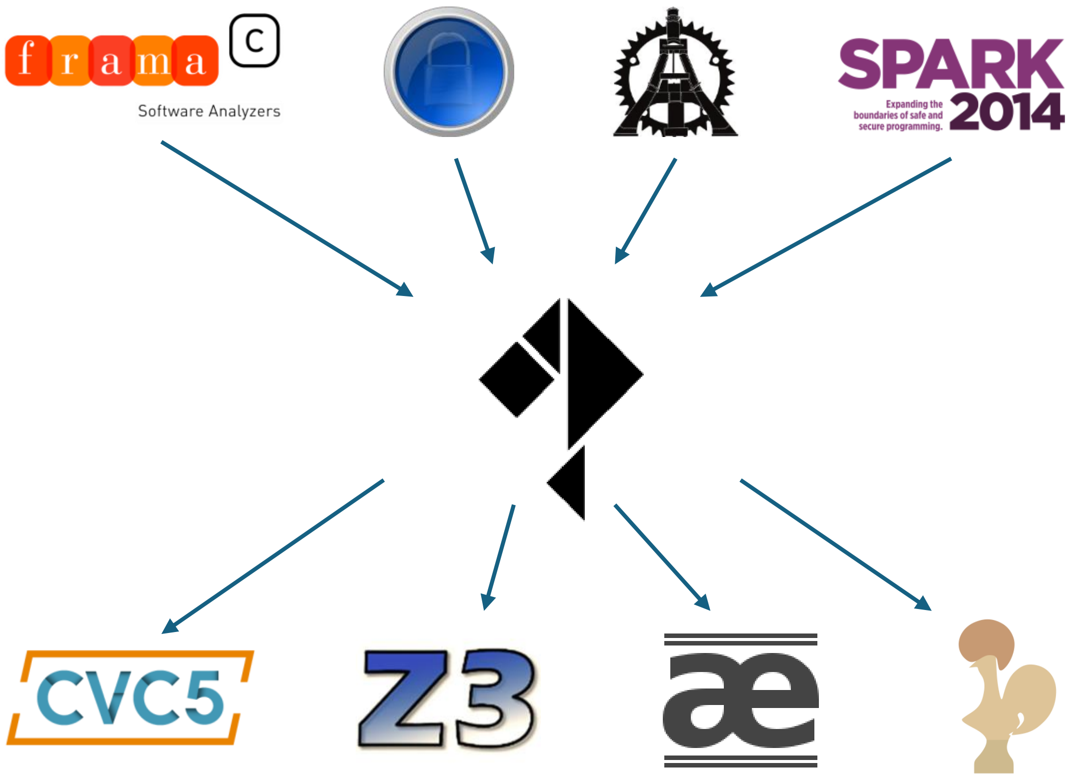

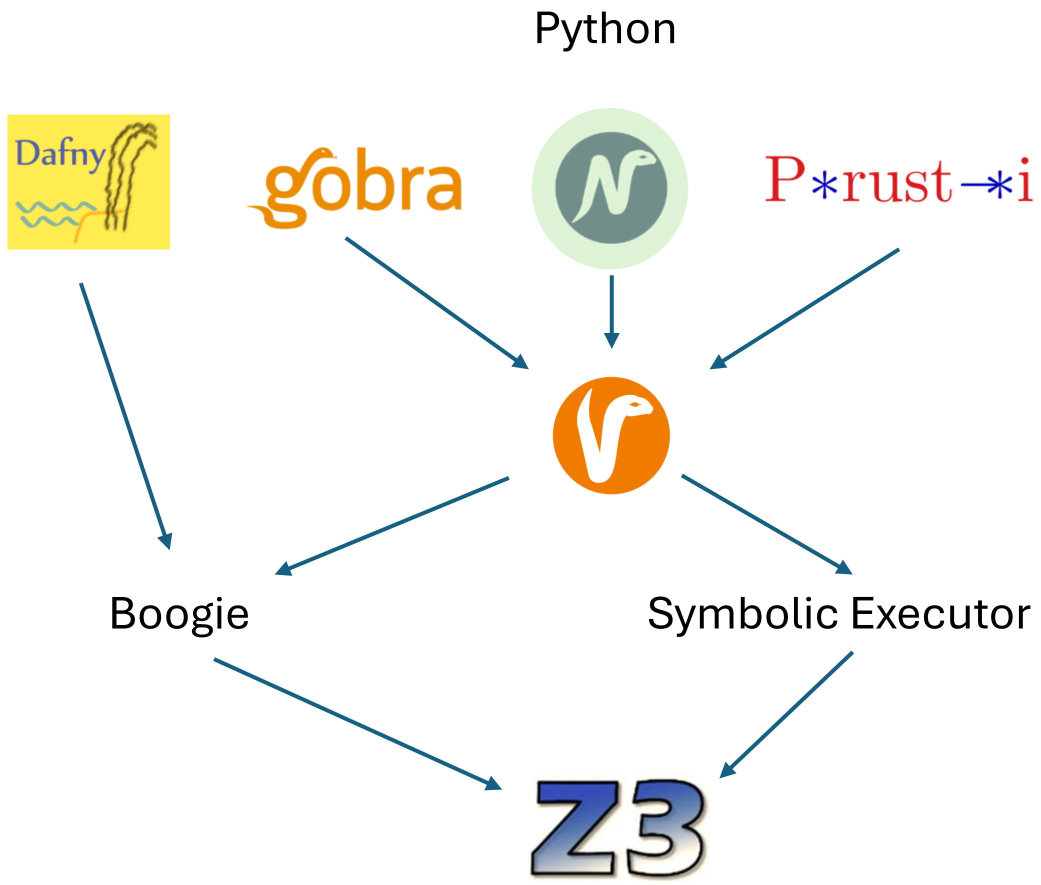

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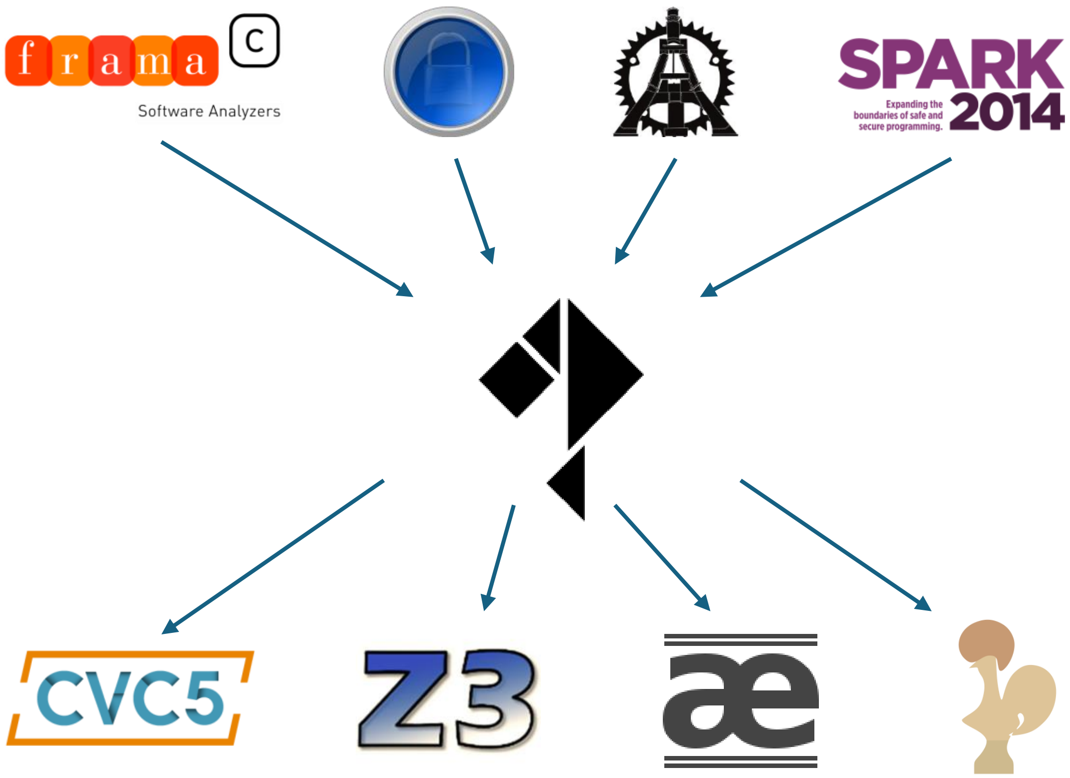

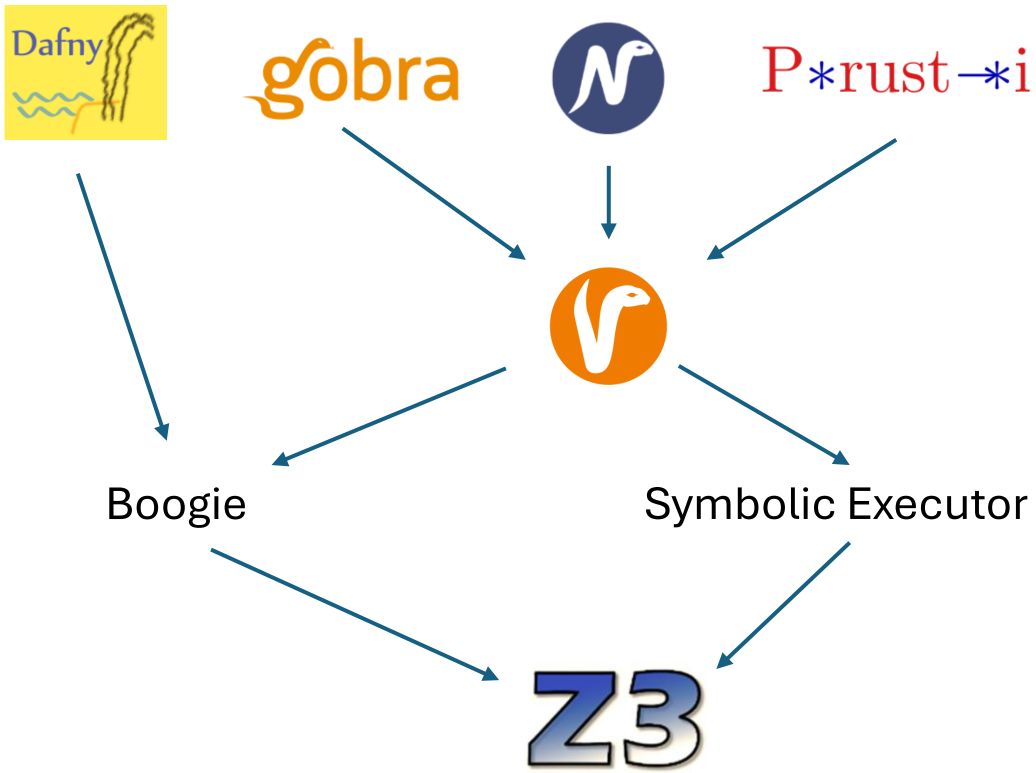

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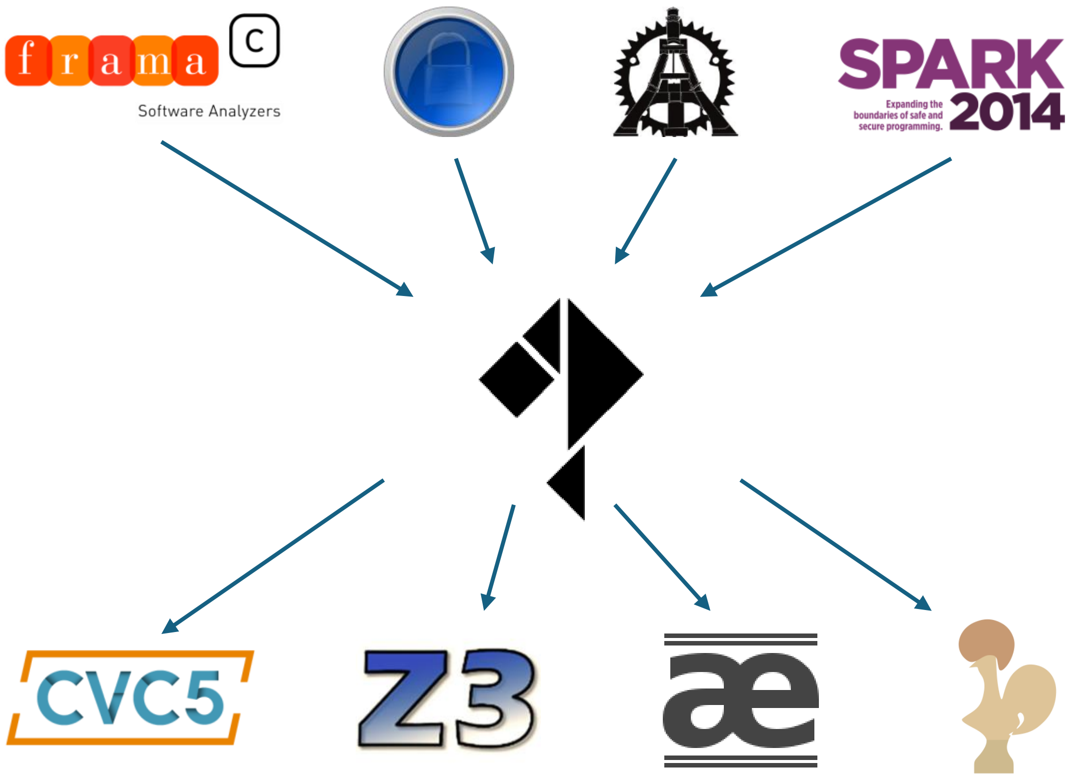

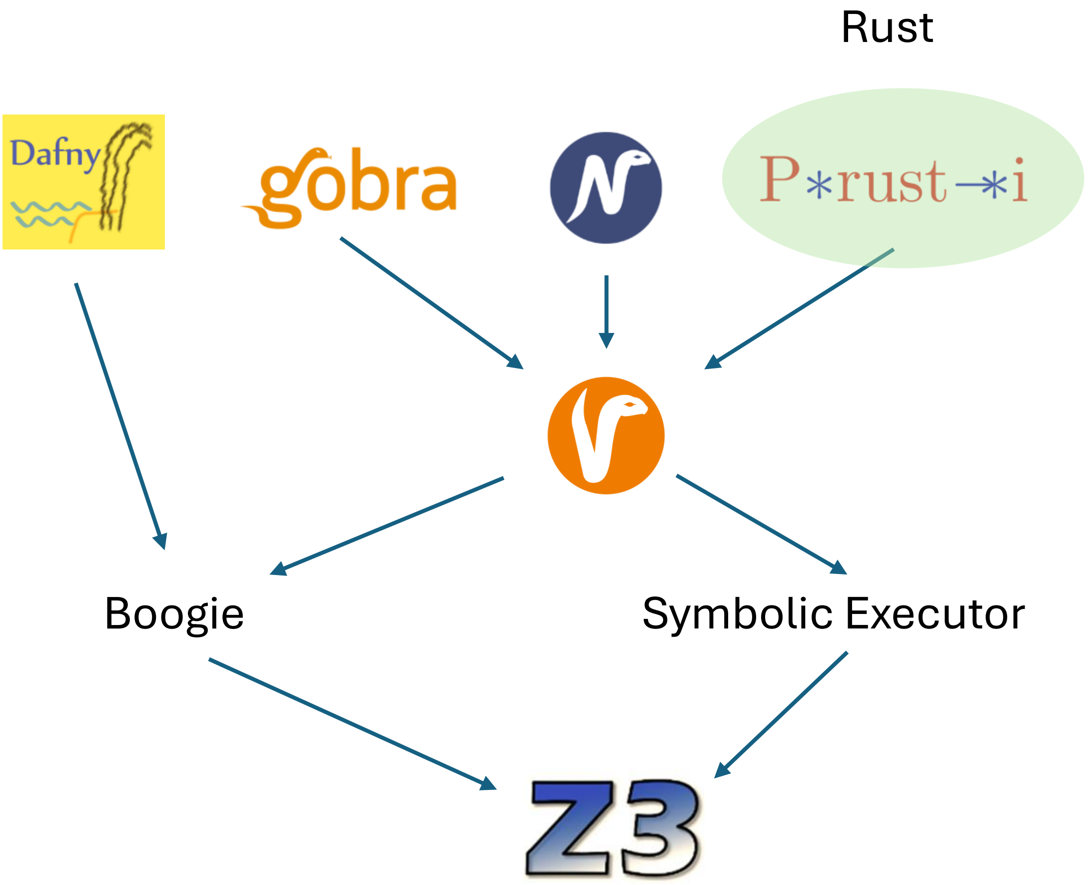

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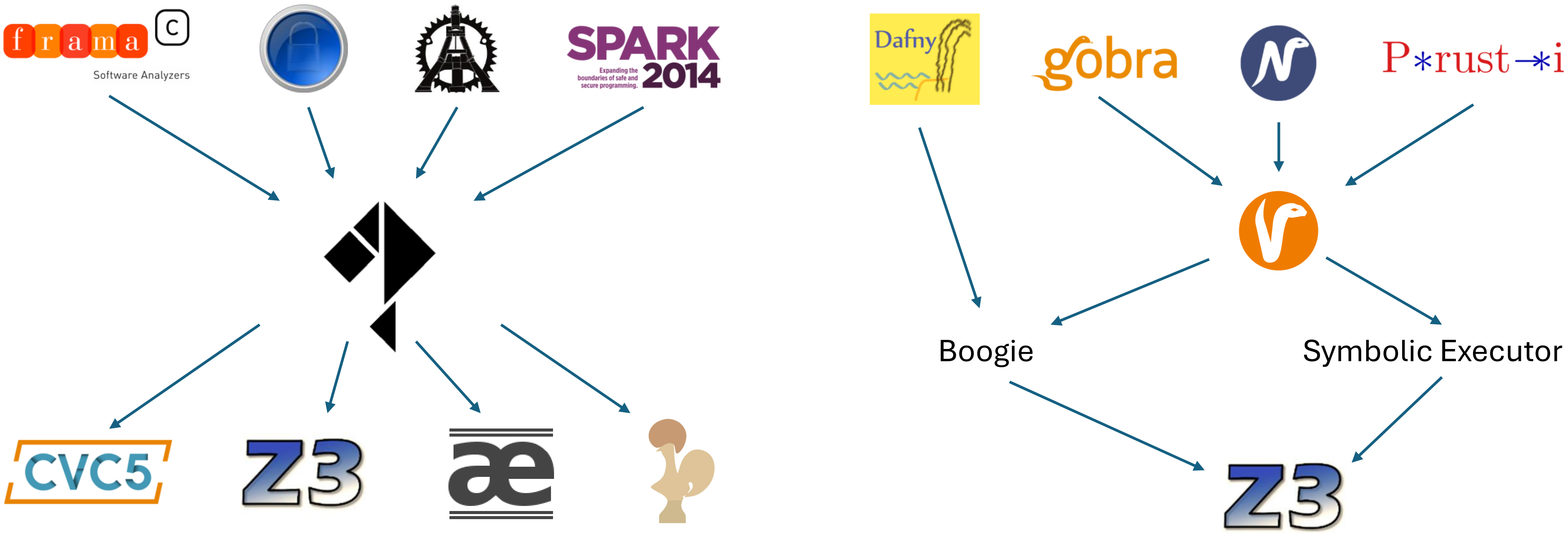
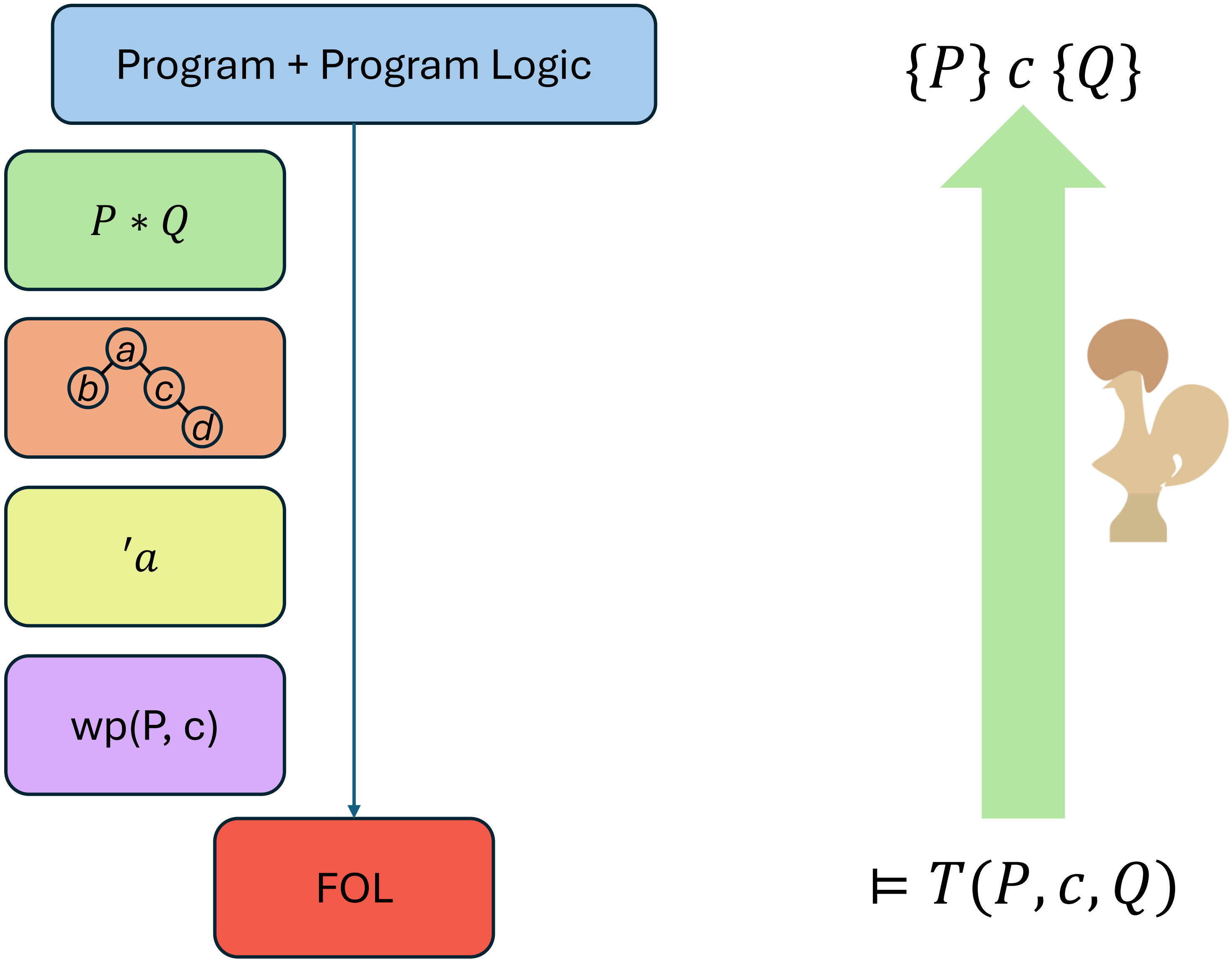


Semi-Automated Verifiers

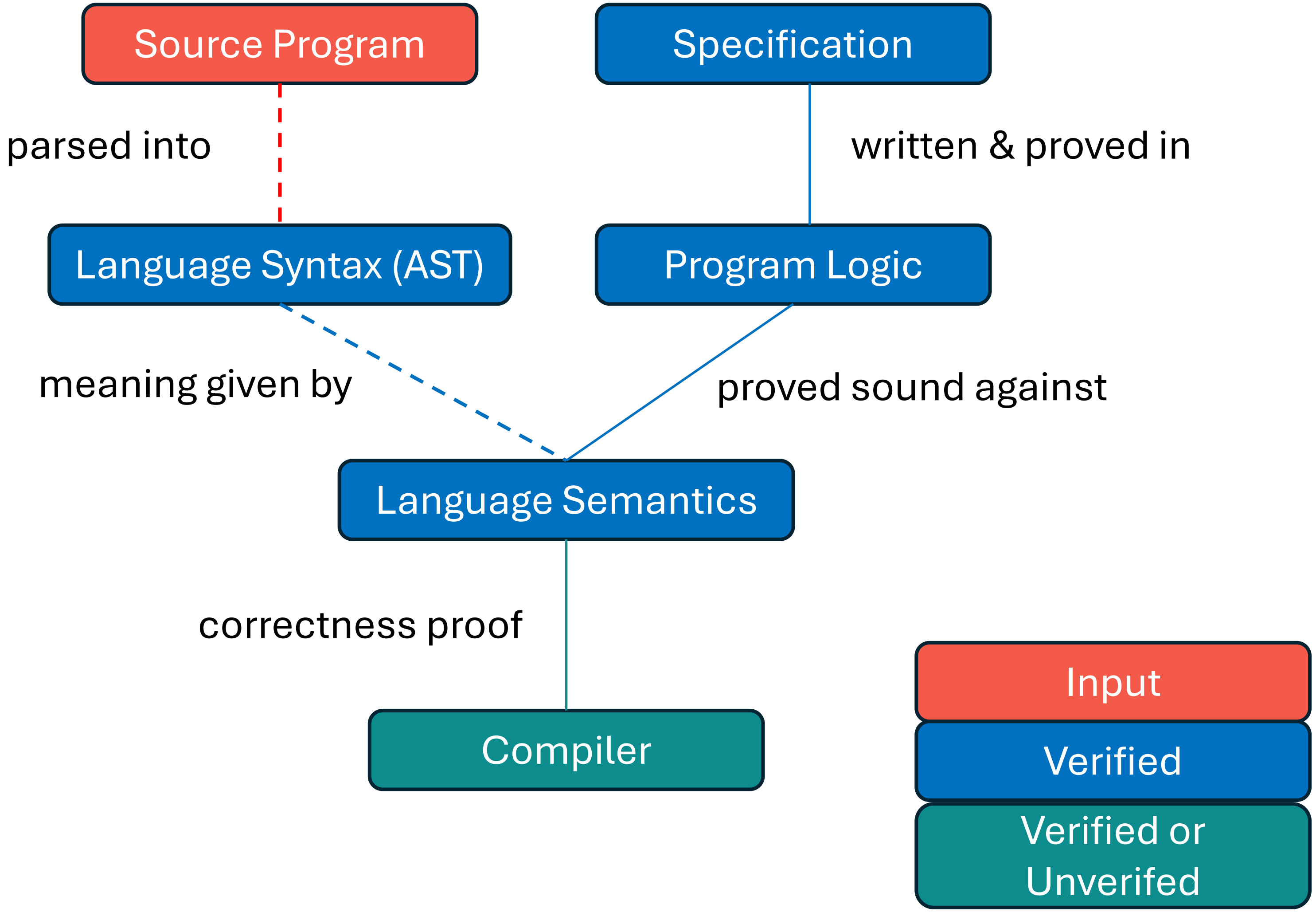
How do we know these tools are correct?

Symbolic Executor

Soundness



Foundational Verifiers



Verified Software Toolchain

CAKEML
A Verified Implementation of ML

Iris

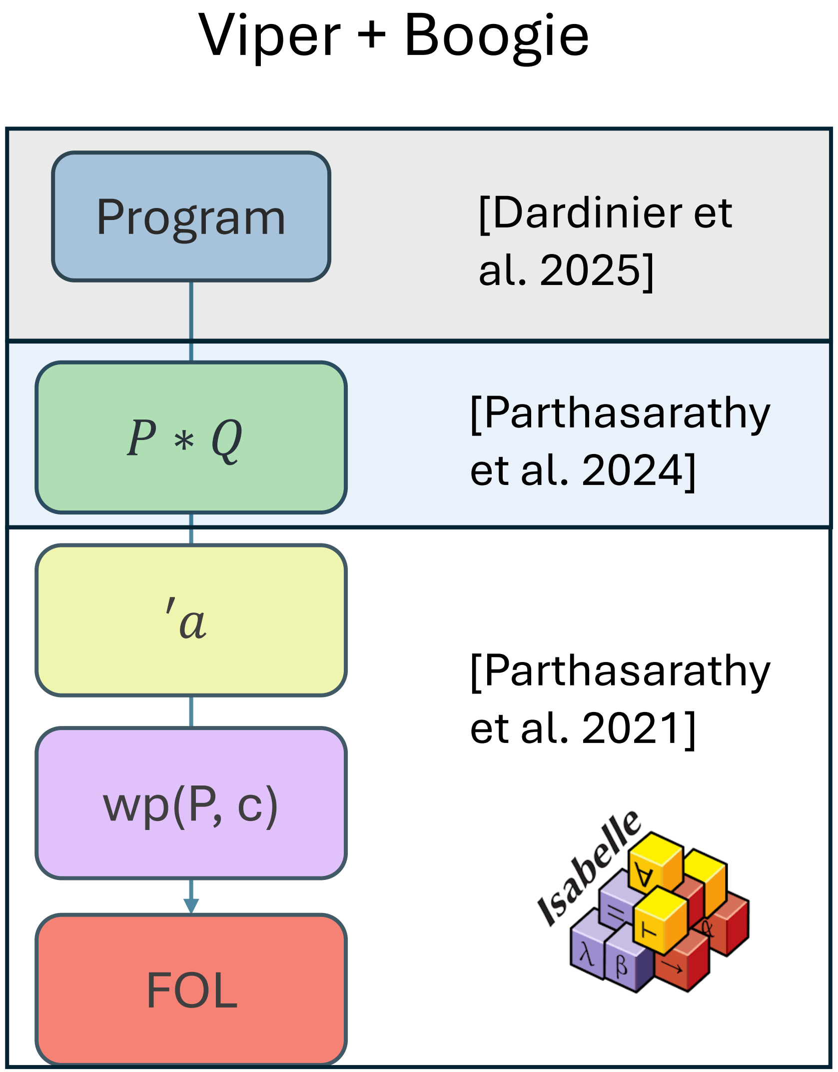
Bedrock2

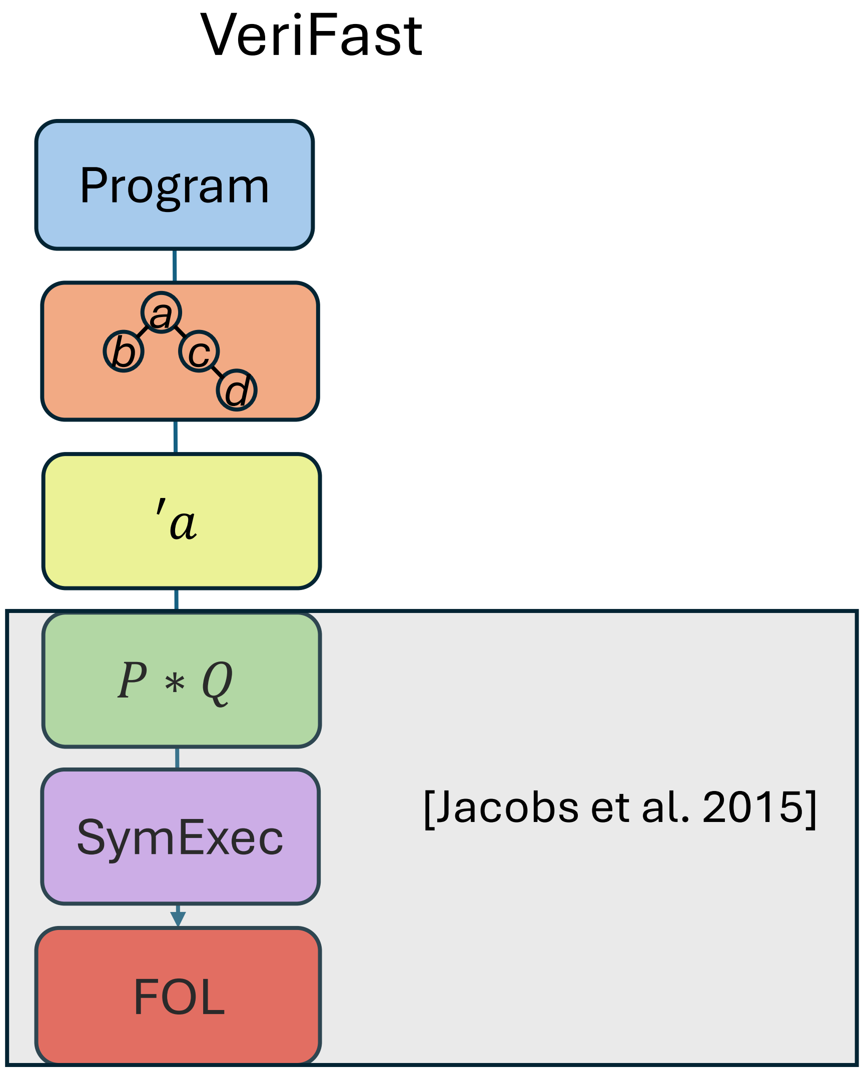
How can we achieve these soundness guarantees for IVL and SMT-based tools?

Not-Quite-Solutions

- Ways to use automated solvers in proof assistants
  - e.g. Sledgehammer [Blanchette et al. 2011], SMTCoq [Ekici et al. 2017], Sniper [Blot et al. 2023], Itauto [Besson 2021], CoqHammer [Czajka and Kaliszyk 2018]
- Improve automation of foundational tools
  - e.g. VST-A [Zhou et al. 2024], RefinedC [Sammler et al. 2021], RefinedRust [Gäher et al. 2024]

Verifying & Validating IVLs





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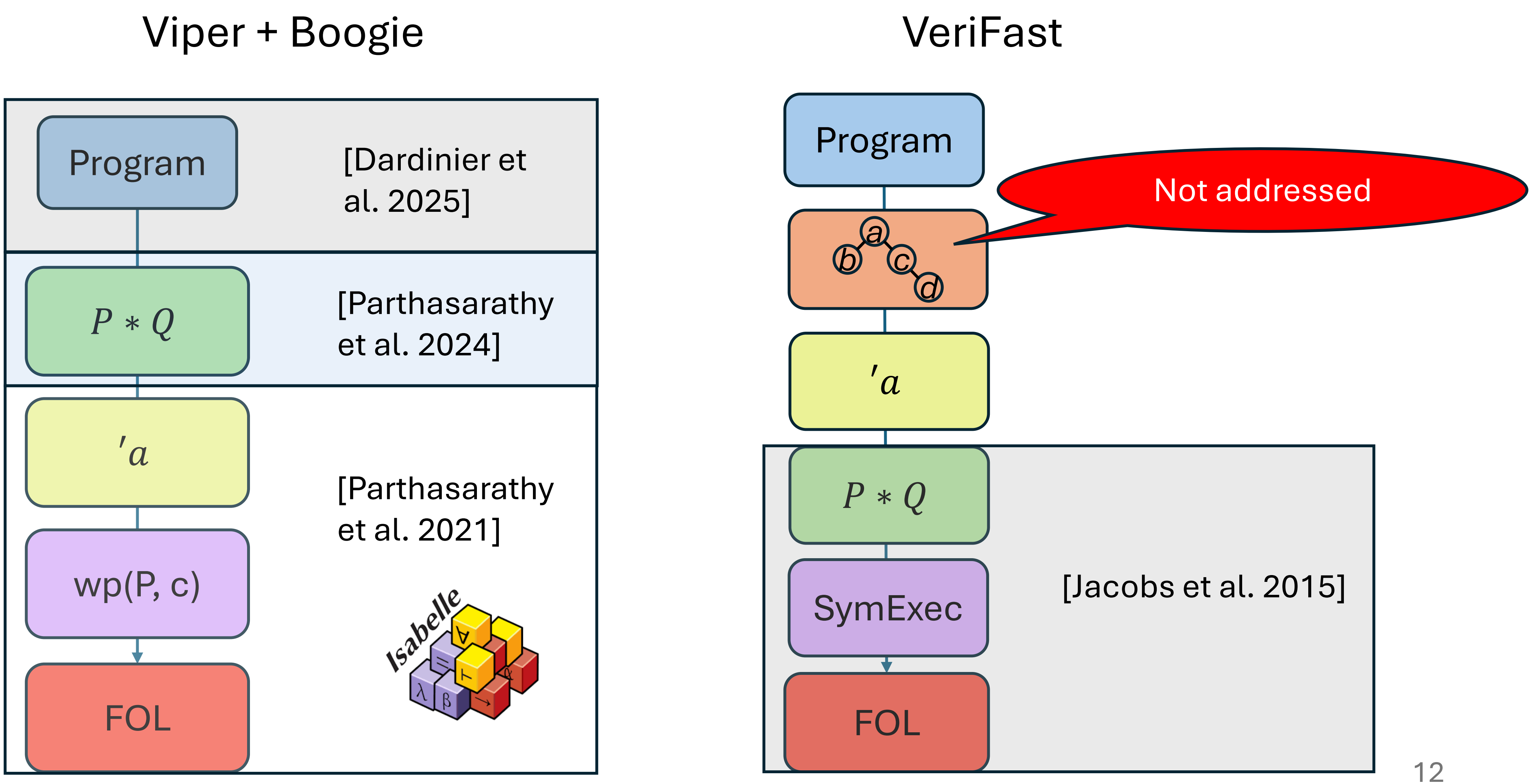

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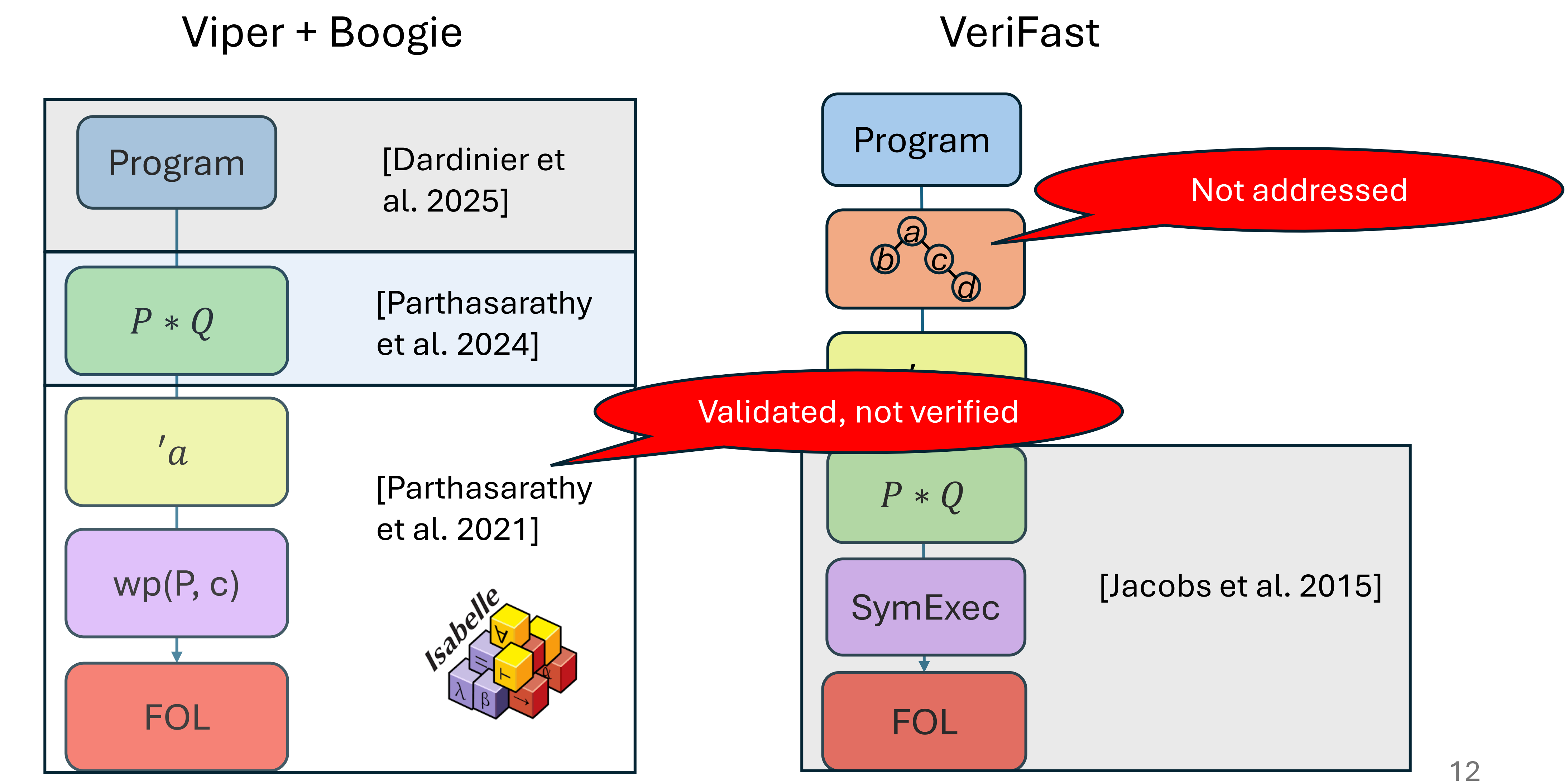

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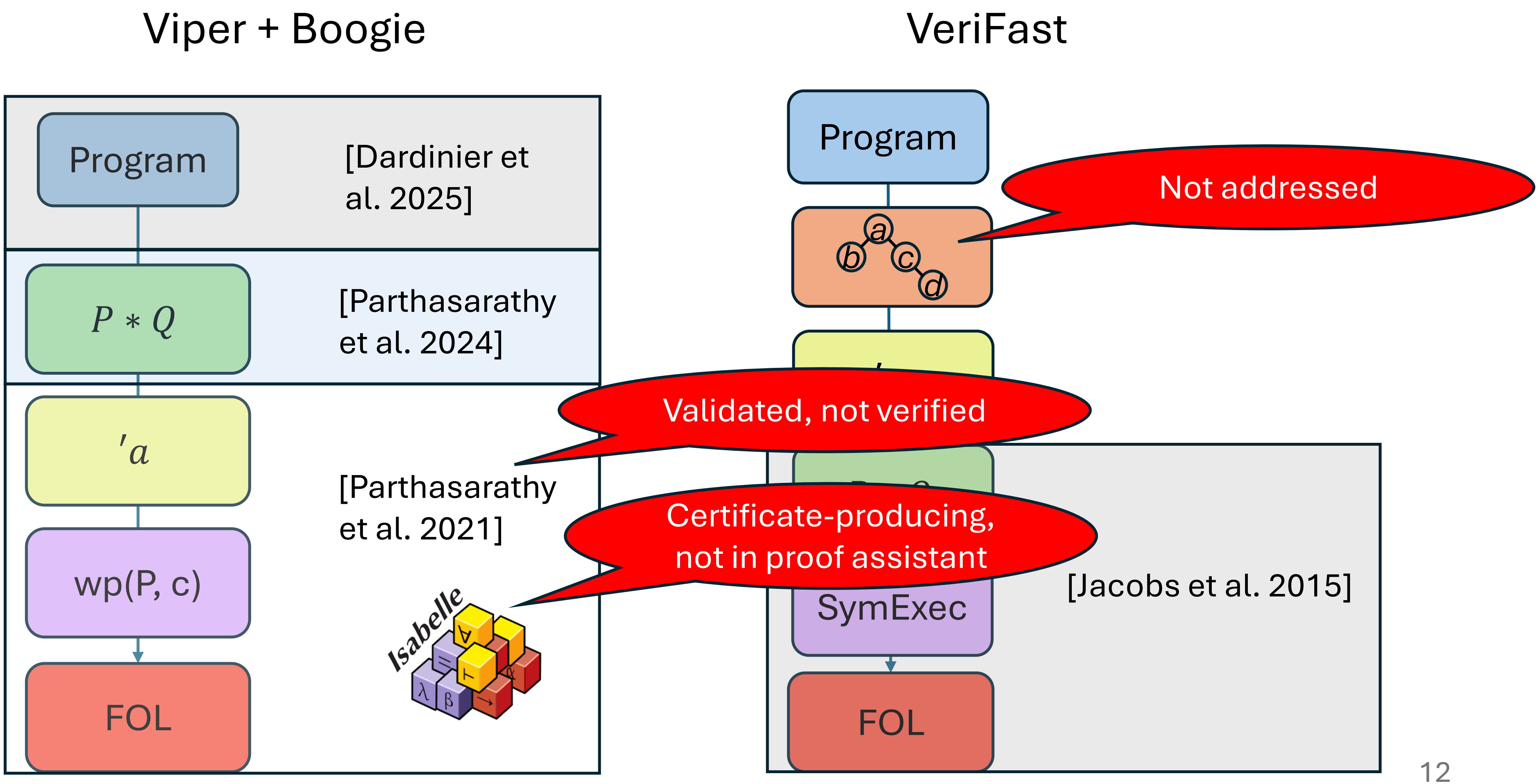


This thesis: a verified implementation of Why3 IVL in Coq

Thesis Contributions

1. A novel formal semantics for Why3's logic
2. A proved-sound compiler from Why3 to (polymorphic) FOL, including
   - Pattern matching compilation
   - Algebraic Data Type axiomatization
3. Why3 API implementation in Coq
   - Method to implement stateful OCaml APIs in Coq
   - Resulting implementation executable both in Coq and OCaml
   - Run existing Why3 + EasyCrypt tests against our tool

Foundational Why3

Coq (Core) | External OCaml

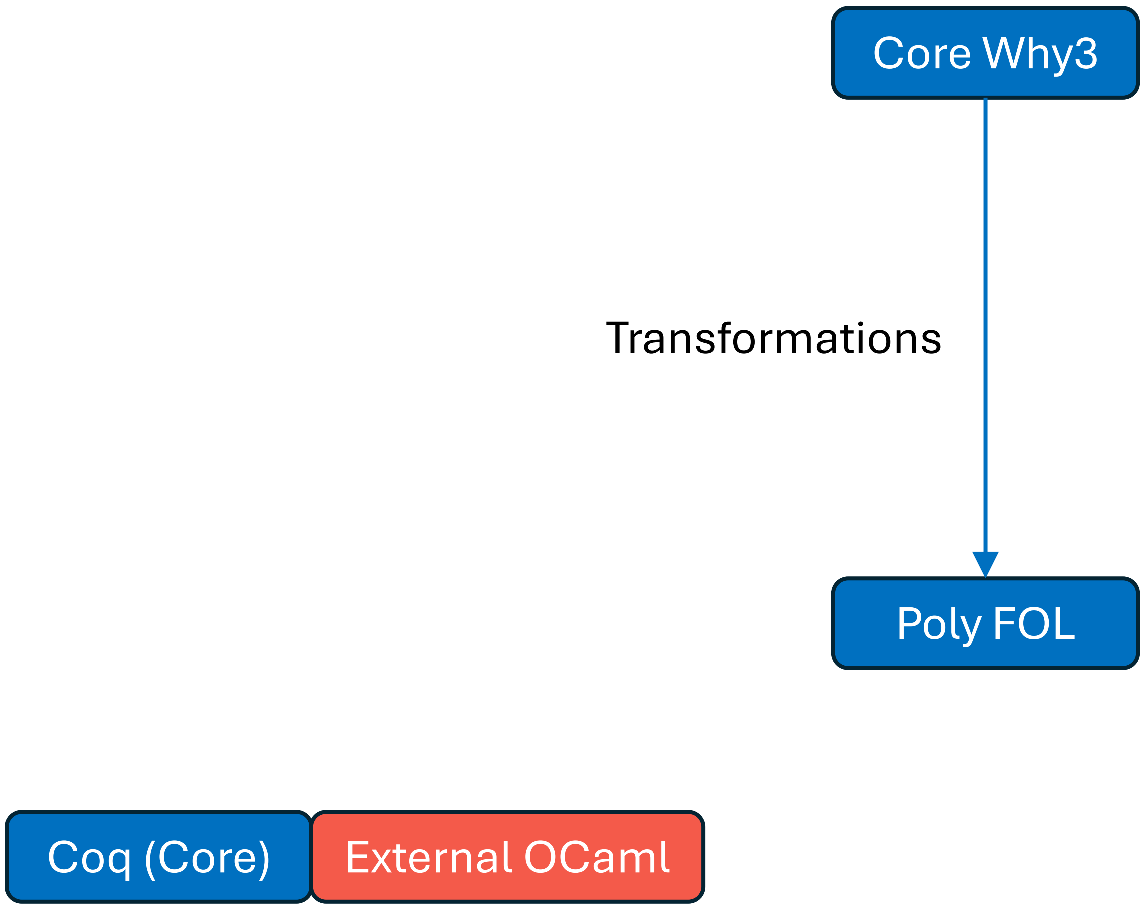
Foundational Why3

Core Why3

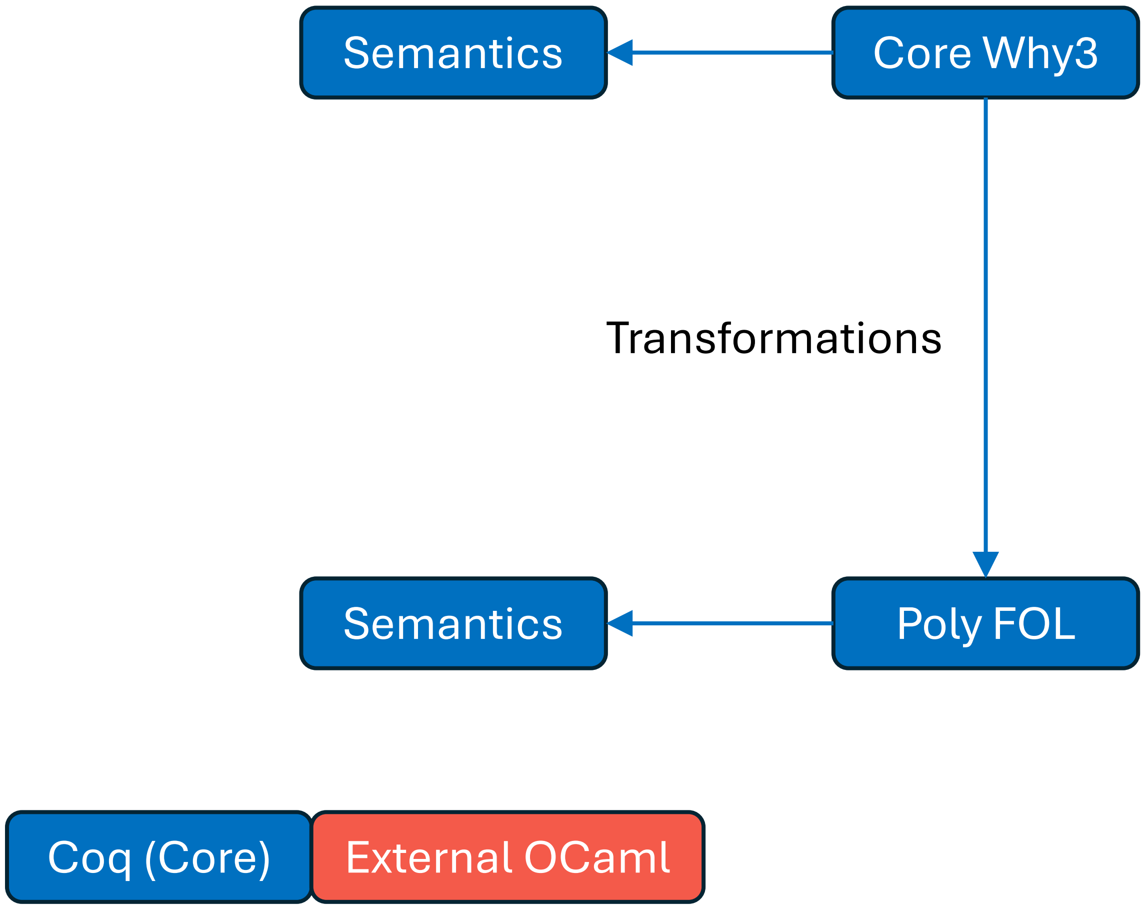
Coq (Core)

External OCaml

Foundational Why3



Foundational Why3



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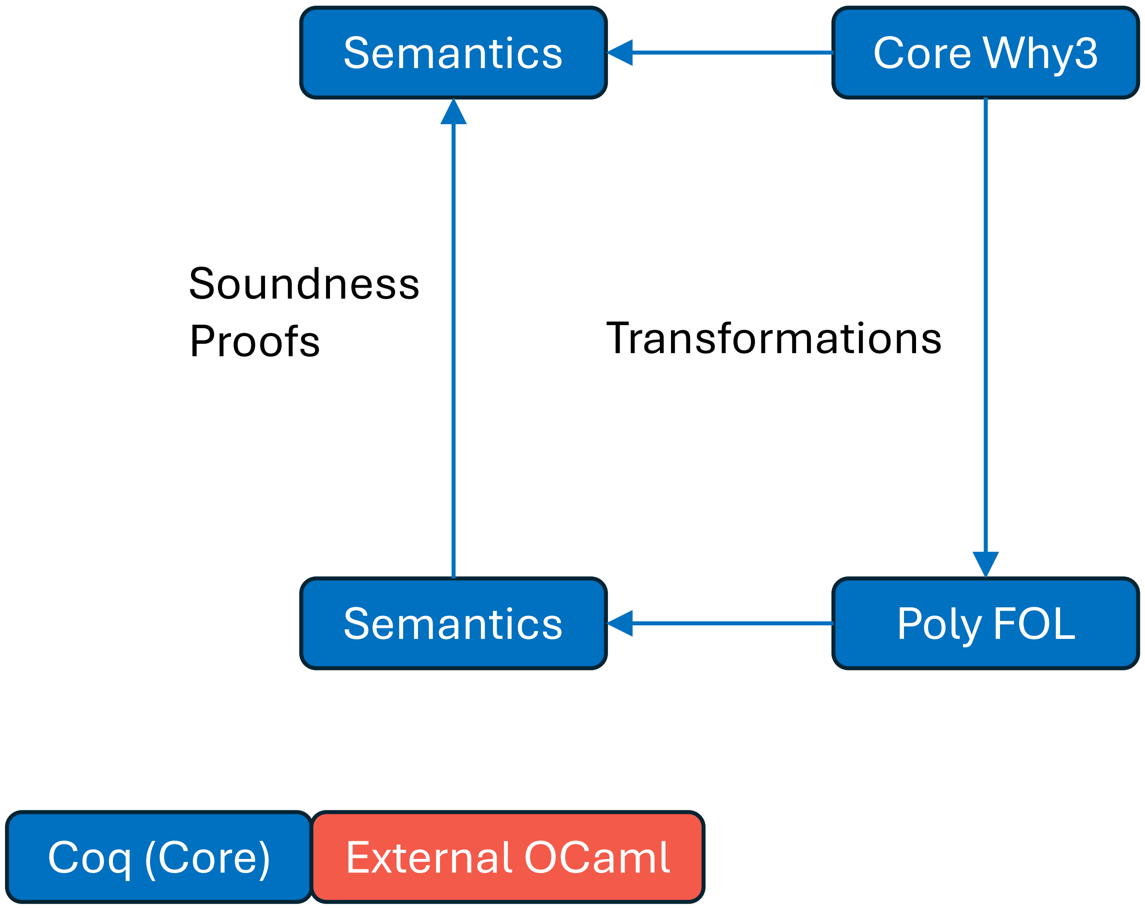

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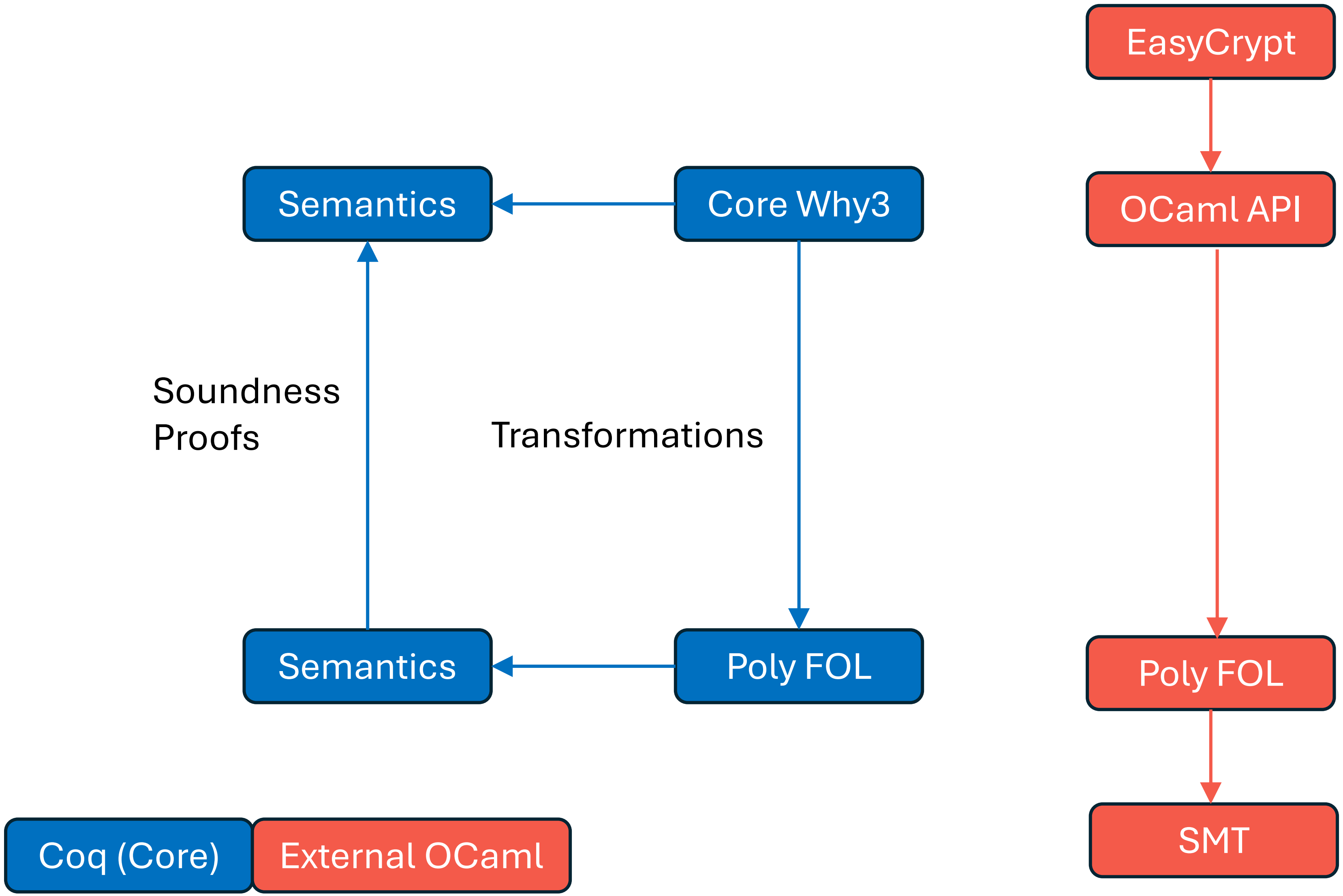

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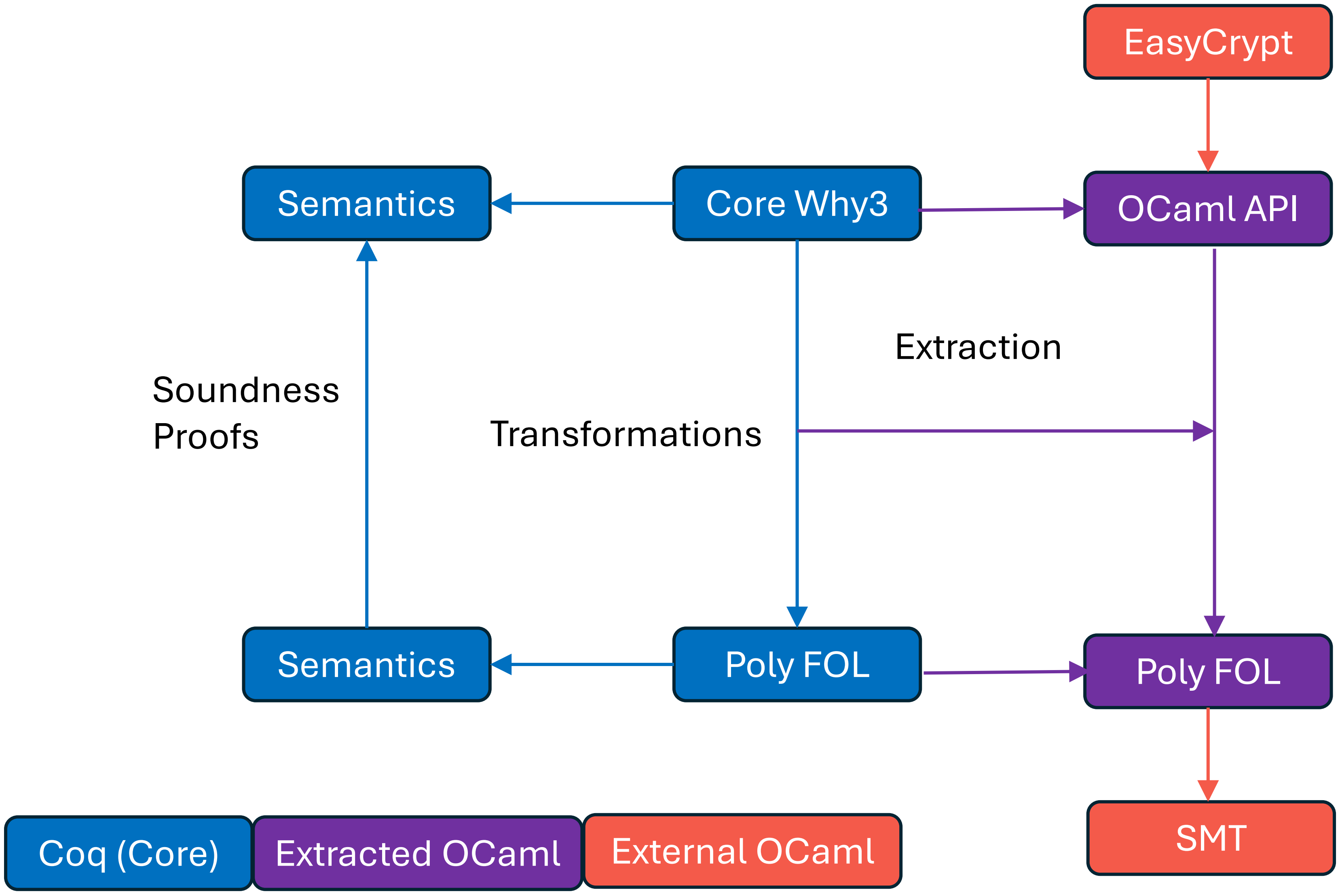

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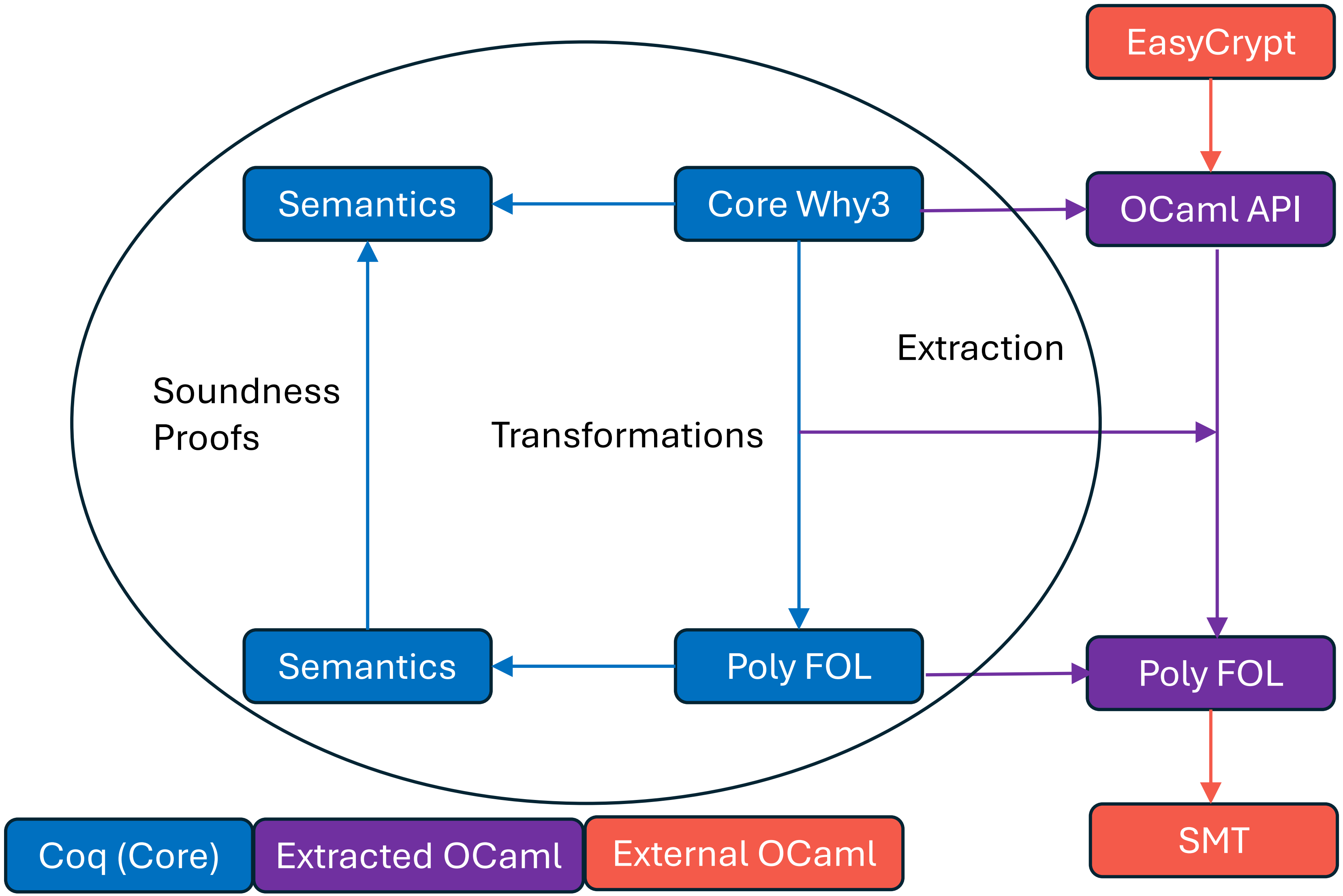
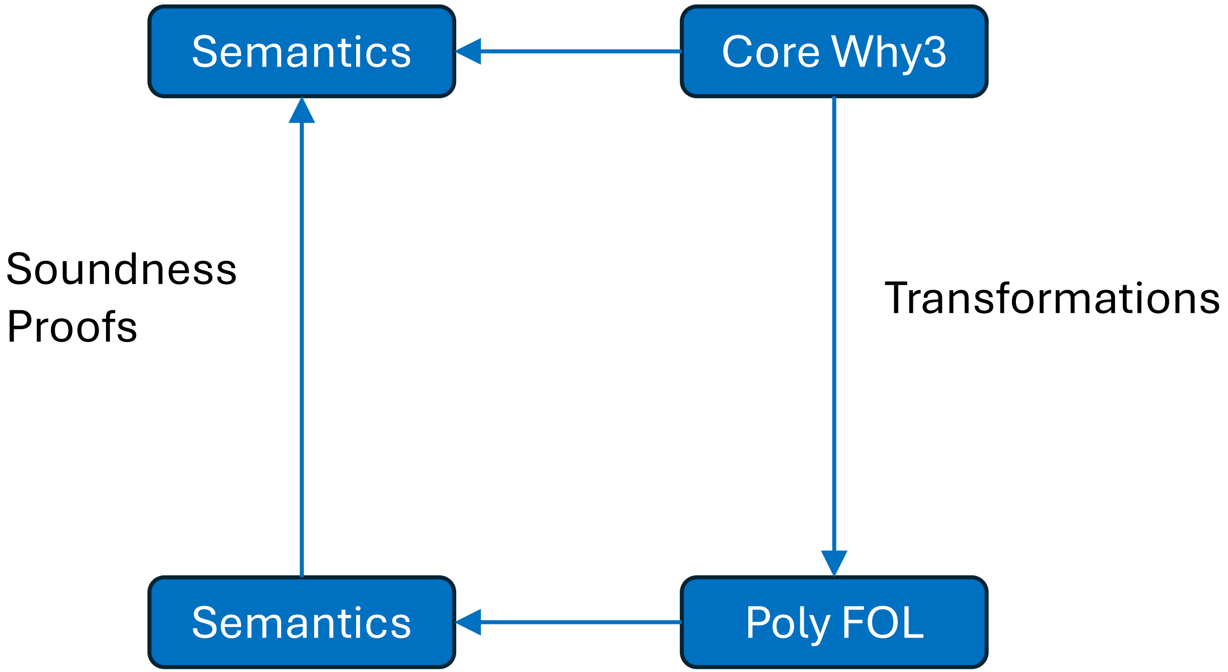


Core Why3


Semantics
Core Why3
Soundness
Proofs
Transformations
Semantics
Poly FOL

Proof Tasks and Entailment

$$(\underbrace{\Gamma}_{\text{Context}}, \underbrace{\Delta}_{\text{Hypotheses}}, \underbrace{g}_{\text{Goal}})$$

Proof Tasks and Entailment

$$(\Gamma, \Delta, g)$$

Context Hypotheses Goal

$$\Gamma, \Delta \vDash g$$

"(Γ, Δ, g) is entailed"

$\coloneqq$*entailed* (Γ, Δ, g)

Transformations

$$T : task \rightarrow list\ task$$

$$\forall\, t, typed(t) \rightarrow \left(\forall\, r\ \in T(t), entailed(r)\right) \rightarrow entailed(t)$$

If all outputs are entailed,

so was the input.

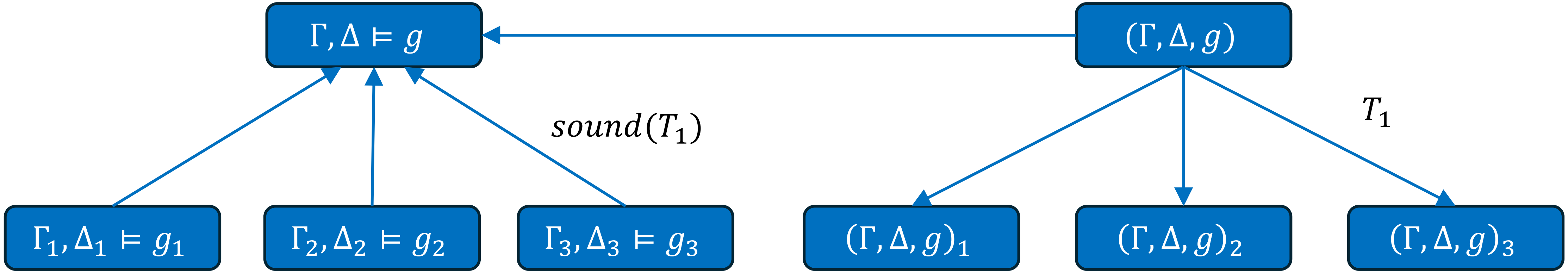
Example

$$T : task \rightarrow list\ task$$

$$\forall\ t, typed(t) \rightarrow \left(\forall\ r\ \in T(t), entailed(r)\right) \rightarrow entailed(t)$$

$$T(\Gamma, \Delta, g_1 \wedge g_2) = \{(\Gamma, \Delta, g_1), (\Gamma, \Delta, g_2)\}$$

Back to Core Why3



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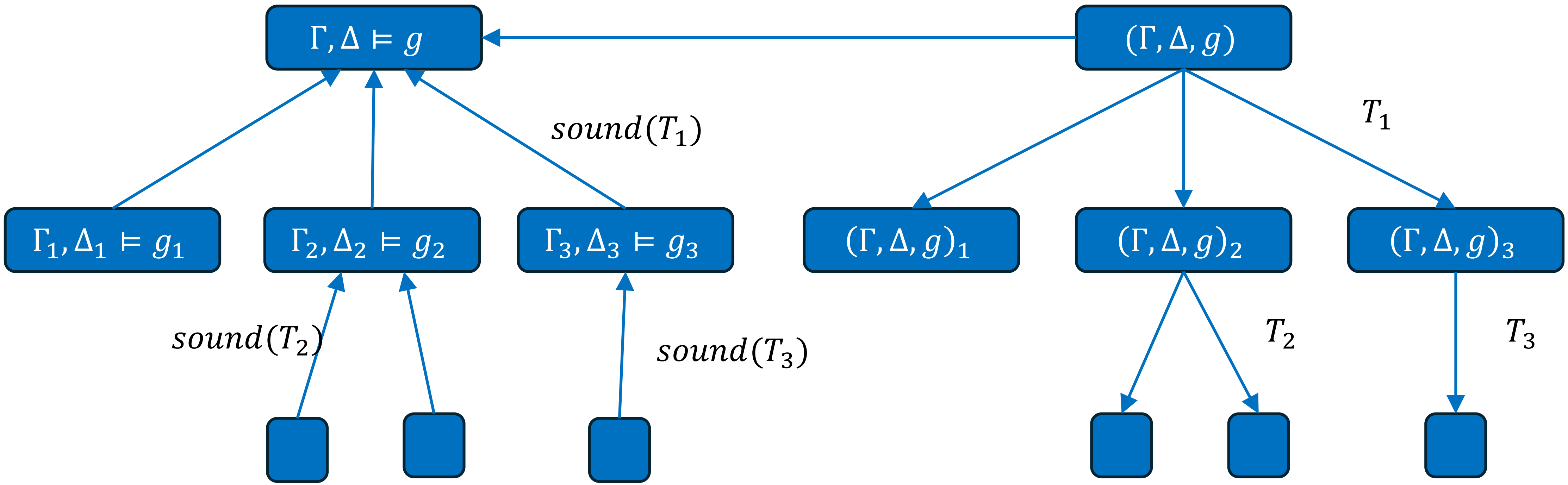

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Cohen, J. M., and Johnson-Freyd, P. A Formalization of Core Why3 in Coq. Proceedings of the ACM on Programming Languages 8, POPL (Jan. 2024), 60:1789–60:1818.

The Logic of Why3

Classical first-order logic with

1. Polymorphism
2. Let- and if-expressions
3. Algebraic data types*
4. Pattern matching
5. Recursive functions and predicates*
6. Inductive predicates*
7. Hilbert's epsilon choice operator

P-FOLDR (**P**olymorphic **F**irst-**O**rder **L**ogic with **D**atatypes and **R**ecursion

```
theory TreeForest
type list 'a = Nil | Cons  'a (list 'a)
type tree 'a = Leaf 'a | Node (tree 'a) (forest 'a)
with forest 'a = list (tree 'a)

use int.Int

function count_forest (f: forest int) : int =
  match f with
  | Nil -> 0
  | Cons t' f' -> count_tree t' + count_forest f'
  end
with count_tree (t: tree int) : int =
  match t with
  | Leaf i -> i
  | Node t' f' -> count_tree t' + count_tree f'
  end
```

Formalizing Why3

- Deep embedding of Why3 in Coq
- Formalize type system
  - Terms and formulas straightforward
  - (Mutual) well-foundedness checks:
    - Recursive function termination
    - Inductive predicate positivity
    - ADT non-emptiness
    - Pattern matching exhaustiveness
- Give verified typechecker

Semantics – Main Ideas

- Hilbert-style (denotational) semantics

$$[g_1 \wedge g_2] = [g_1] \wedge [g_2]$$

Why3 "and"

Meta-logic (Coq) "and"

- Give *model* of Why3 in Coq

Semantics – Main Ideas

- Hilbert-style (denotational) semantics

Interpretation of type, function, predicate symbols

Valuation of type and term variables

$$[g_1 \wedge g_2]_{I,v} = [g_1]_{I,v} \wedge [g_2]_{I,v}$$

Why3 “and”

Meta-logic (Coq) “and”

- *Truth* given by fixed interpretation and valuation
- *Validity* – true under all interpretations

Semantics: Types

$[f]_v : Prop$ for formula f

$[t]_v : [v(\tau)]$ for term t of Why3 type τ

Return type *depends* on Why3 type, interpretation, and valuation!

Semantics - Examples

$$[x]_v = v(x)$$

$$[\forall x, g]_v = \forall d, [g]_{v[x \to d]}$$

$$[f(\tau_1, \tau_2, \dots, \tau_m)(t_1, t_2, \dots t_n)]_v$$
$$= \left[f\left(v(\tau_1), v(\tau_2), \dots, v(\tau_m)\right)\right]_I([t_1]_v, \dots, [t_n]_v)$$

Function interpretation with type substitution

Heterogenous list of arguments

Semantics – Recursive Structures

Prior pen-and-paper description [Filliâtre 2013] merely imposes conditions on interpretation:

An *interpretation* is a pre-interpretation that is consistent with recursive and inductive definitions, that is:

- For any recursive definition `function` $\mathsf{f}\langle\boldsymbol{\alpha}\rangle(\boldsymbol{x}) : \tau = t$ and any $\boldsymbol{s}$, we require $[\![\mathsf{f}\langle\boldsymbol{s}\rangle]\!]$ to be such that, for all $\boldsymbol{t}$, $[\![\mathsf{f}\langle\boldsymbol{s}\rangle]\!](\boldsymbol{t}) = [\![t]\!]_v$ where v maps the $\boldsymbol{\alpha}$ to the $\boldsymbol{s}$ and the $\boldsymbol{x}$ to the $\boldsymbol{t}$ (and similarly for a predicate definition).
- For any inductive definition $\mathsf{p}\langle\boldsymbol{\alpha}\rangle(\boldsymbol{\tau}) = f_1 \mid \ldots \mid f_l$ and any $\boldsymbol{s}$, we require $[\![\mathsf{p}\langle\boldsymbol{s}\rangle]\!]$ to be the least predicate such that $[\![f_1]\!]_v, \ldots, [\![f_l]\!]_v$ hold where v maps the $\boldsymbol{\alpha}$ to the $\boldsymbol{s}$.

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How do we know that such interpretations (models) exist?
If not, every formula is valid!

Recursive Structures

We give explicit, generic constructions satisfying these properties:

- ADTs → **W-types**
- Inductive predicates → **impredicative encoding**
- Recursive functions → **well-founded recursion**

Users of semantics need only properties, not complex encodings

Algebraic Data Types

1. Constructors are injective: if $[c](t_1) = [c](t_2)$, then $t_1 = t_2$
2. Constructors are disjoint: if $[c_1](t_1) = [c_2](t_2)$, then $c_1 = c_2$
3. There is a (computable) function $\boldsymbol{find}$ that gives the constructor c and arguments t for any element x of ADT type such that
$$x = [c](t)$$
4. A generalized induction principle holds

Implement ADTs using W-types, prove these properties satisfied

Pattern matching: describe new bound variables, use $\boldsymbol{find}$ for constructors

Recursive Functions

- Relies on many parts of typing and semantics:

1. Define well-founded relation on W-type encoding denoting structural inclusion
2. Prove (via induction on ADT interpretation) that pattern matching produces “smaller” variables
3. Use termination check to show recursion occurs on variables from pattern match

Entailment Revisited

Entailment Revisited

$$\Gamma, \Delta \vDash g$$

Entailment Revisited

$$\Gamma, \Delta \vDash g$$

$$\forall I, full(I) \rightarrow \left(\forall d\ v, d \in \Delta \rightarrow [d]_{I,v}\right) \rightarrow (\forall v, [g]_{I,v})$$

Entailment Revisited

$$\Gamma, \Delta \vDash g$$

$$\forall I, full(I) \to \left(\forall d\ v, d \in \Delta \to [d]_{I,v}\right) \to (\forall v, [g]_{I,v})$$

For every interpretation I consistent with defined types, functions, and predicates,

Entailment Revisited

$$\Gamma, \Delta \vDash g$$

$$\forall I, full(I) \to \left(\forall d\ v, d \in \Delta \to [d]_{I,v}\right) \to (\forall v, [g]_{I,v})$$

For every interpretation I consistent with defined types, functions, and predicates,

If every formula in Δ is satisfied by I,

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For every interpretation I consistent with defined types, functions, and predicates,

If every formula in Δ is satisfied by I,

Then g is satisfied by I.

P-FOLDR as a Logic

Consistency:

```
Theorem consistent (pd: pi_dom) (pdf: pi_dom_full gamma pd)
(pf: pi_funpred gamma_valid pd pdf)
(pf_full: full_interp gamma_valid pd pf) (f: formula)
(f_typed: formula_typed gamma f):
~ (satisfies pd pdf pf pf_full f f_typed /\
  satisfies pd pdf pf pf_full (Fnot f) (F_Not f_typed)).
```

Existence of models:

```
Theorem full_interp_exists: forall funi predi,
  {pf: pi_funpred gamma_valid pd pdf |
    full_interp gamma_valid pd pf /\
    (forall f srts a, In (abs_fun f) gamma ->
      (funs gamma_valid pd pf ) f srts a = funi f srts a) /\
    (forall p srts a, In (abs_pred p) gamma ->
      (preds gamma_valid pd pf) p srts a = predi p srts a)}.
```

A Sound Proof System for P-FOLDR

- Build sound-by-construction, natural-deduction-style proof system and tactic system
- Prove all introduction and elimination rules for connectives
- Prove rules for induction, unfolding recursive functions, type substitution, rewriting, etc
- Prove lemmas from Why3's standard library about lists and trees, e.g.

```
lemma inorder_length: ∀ t : tree 'a, length (inorder t) = size t
```

- Gives confidence that our semantics matches the intended one

Thesis Contributions

1. A novel formal semantics for Why3's logic
2. **A proved-sound compiler from Why3 to (polymorphic) FOL, including**
    - **Pattern matching compilation**
    - Algebraic Data Type axiomatization
3. Why3 API implementation in Coq
    - Method to implement stateful OCaml APIs in Coq
    - Resulting implementation executable both in Coq and OCaml

Pattern Matching in P-FOLDR

$$
\begin{aligned}
p \coloneqq \quad & \mid \_ \\
& \mid x \\
& \mid c(p_1, \ldots, p_n) \\
& \mid p_1 | p_2 \\
& \mid p \; as \; x
\end{aligned}
$$

Pattern Matching in P-FOLDR

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Complicated!

- Nested matching
- Simultaneous matching
- Interactions with termination checking

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\end{aligned}
$$

Complicated!

- Nested matching
- Simultaneous matching
- Interactions with termination checking

We defined pattern/matching

- Syntax
- Typing
- Semantics

Semantics based on describing valuations of newly bound variables

Pattern Matching Compilation

```
match l1, l2 with
| [], [] -> x1
| [_], _ -> x2
| _ :: _, _ -> x3
| [], _ :: _ -> x4
end
```

```
match l1 with
| [] -> match l2 with
        | [] -> x1
        | y3 :: y4 -> x4
        end
| y1 :: y2 ->
        match y2 with
        | [] -> x2
        | y5 :: y6 -> x3
        end
end
```

Pattern Matching Compilation

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match l1, l2 with
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        end
end
```

```
match l1, l2 with
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| [_], _ -> x2
| _ :: _, _ -> x3
end
```

Non-exhaustive!

Pattern Matching Compilation

- Widely applicable and well-studied problem: e.g. OCaml, Haskell, Coq

[Augustsson 1985], [Baudinet and MacQueen 1985], [Laville 1988], [Puel and Suarez 1990], [Maranget 1992], [Pettersson 1992], [Sekar et al. 1995], [Sestoft 1996], [Scott and Ramsey 2000], [Le Fessant and Maranget 2001], [Maranget 2007], [Maranget 2008], [Karachalias 2015], [Tuerk et al. 2015]

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Pattern Matching Compilation [Le Fassant and Maranget 2001, Maranget 2008]

```
match l1, l2 with
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```

$$\begin{pmatrix} nil & nil & x_1 \\ cons(\_, nil) & nil & x_2 \\ cons(\_, \_) & \_ & x_3 \\ nil & cons(\_, \_) & x_4 \end{pmatrix}$$

Pattern Matching Compilation [Le Fassant and Maranget 2001, Maranget 2008]

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$$\begin{pmatrix} nil & nil & x_1 \\ cons(\_, nil) & nil & x_2 \\ cons(\_,\_) & \_ & x_3 \\ nil & cons(\_,\_) & x_4 \end{pmatrix}$$

$S(c, P)$: rest of match, assuming the first term matches constructor c

$$S(nil, P) = \begin{pmatrix} nil & x_1 \\ cons(\_,\_) & x_4 \end{pmatrix} \qquad S(cons, P) = \begin{pmatrix} \_ & nil & nil & x_2 \\ \_ & \_ & \_ & x_3 \end{pmatrix}$$

1. Termination

- Recurse on decompositions, not subterms
- “or” patterns and added wildcards make matrix larger
- Prove generic termination results for any matrix-based algorithm

2. Semantics

If $compile(P, ts) = Some\ t$, then $\left[[ts]_v, P\right]_v = Some\ [t]_v$

- Relies on semantics of pattern matrix matching
- Purely *semantic* reasoning – need to reason about ADTs, $\boldsymbol{find}(x)$, interpretation
- Existing proofs in literature – based on *syntactic* match relation on values

$$c(v_1, \dots, v_n) \preccurlyeq c(p_1, \dots, p_n) \leftrightarrow \forall i, v_i \preccurlyeq p_i$$

- Different than our setting: match semantics depends on interpretation!

3. Exhaustiveness

Corollary of semantic correctness:

$$\text{If } \left[[ts]_v, P\right]_v = None, \text{ then } compile(P, ts) = None$$

- Different than proofs in the literature [Maranget 2007]
- Adapt proofs for purely logical setting vs call-by-value or lazy evaluation

4. Robustness

- Exhaustiveness check succeeds under reasonable changes to types, terms, patterns, etc
  - E.g. substitution, alpha-conversion, rewriting, etc
  - Not true in Why3! Rewriting can cause exhaustiveness check to fail
  - Found and reported bug to Why3 developers
  - Fixed with provably stronger exhaustiveness check

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   - Method to implement stateful OCaml APIs in Coq
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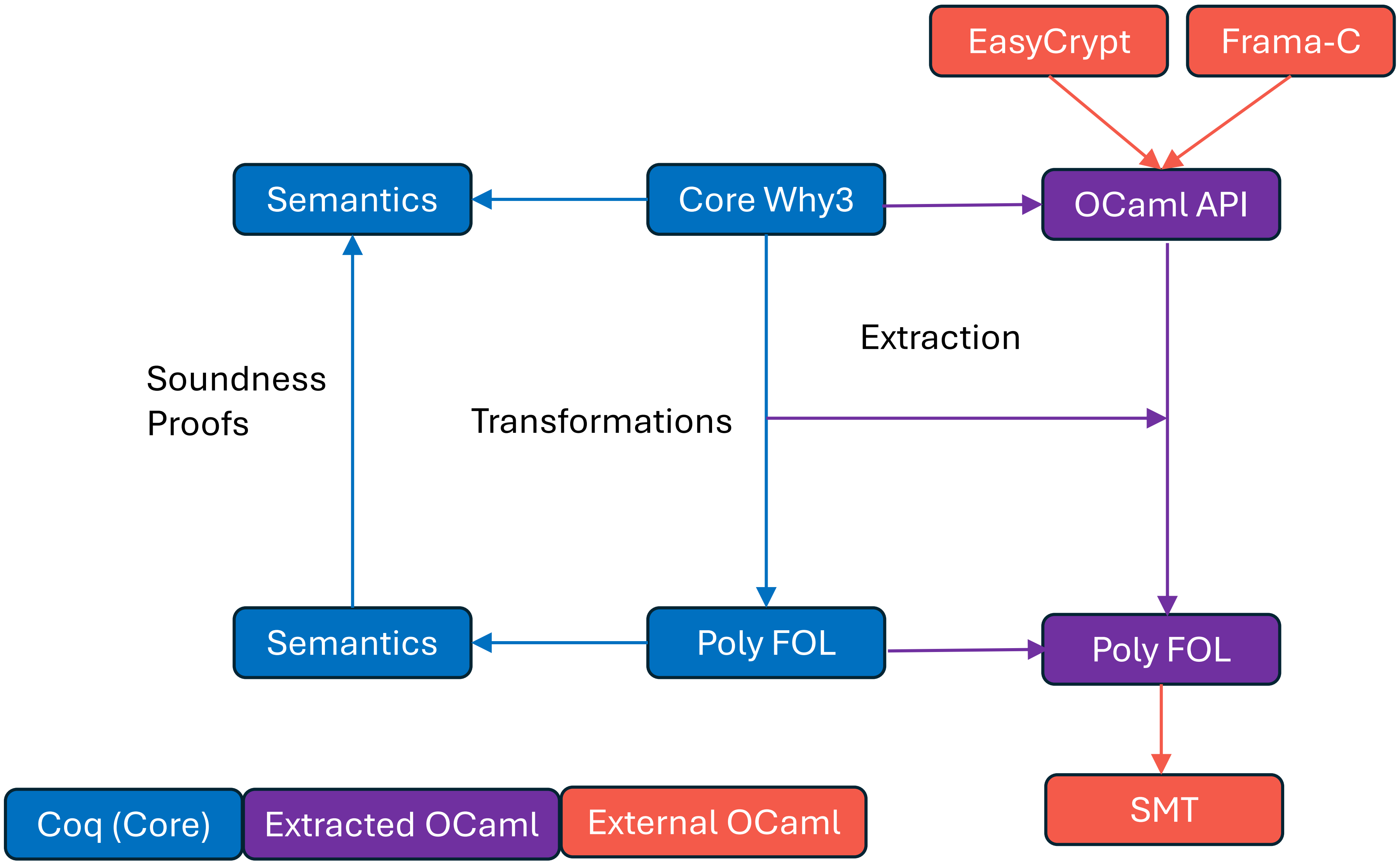
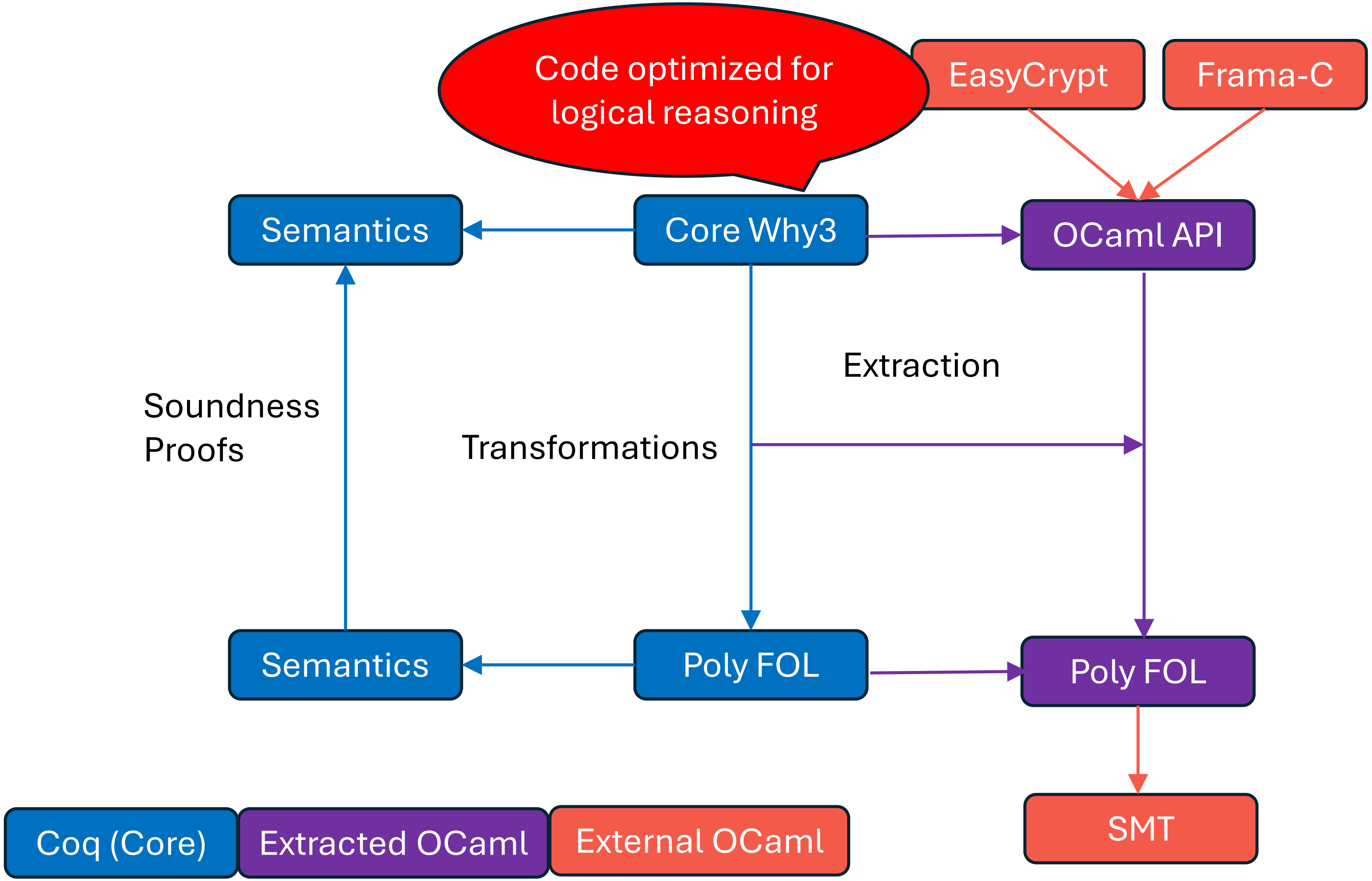
Foundational Why3



Foundational Why3



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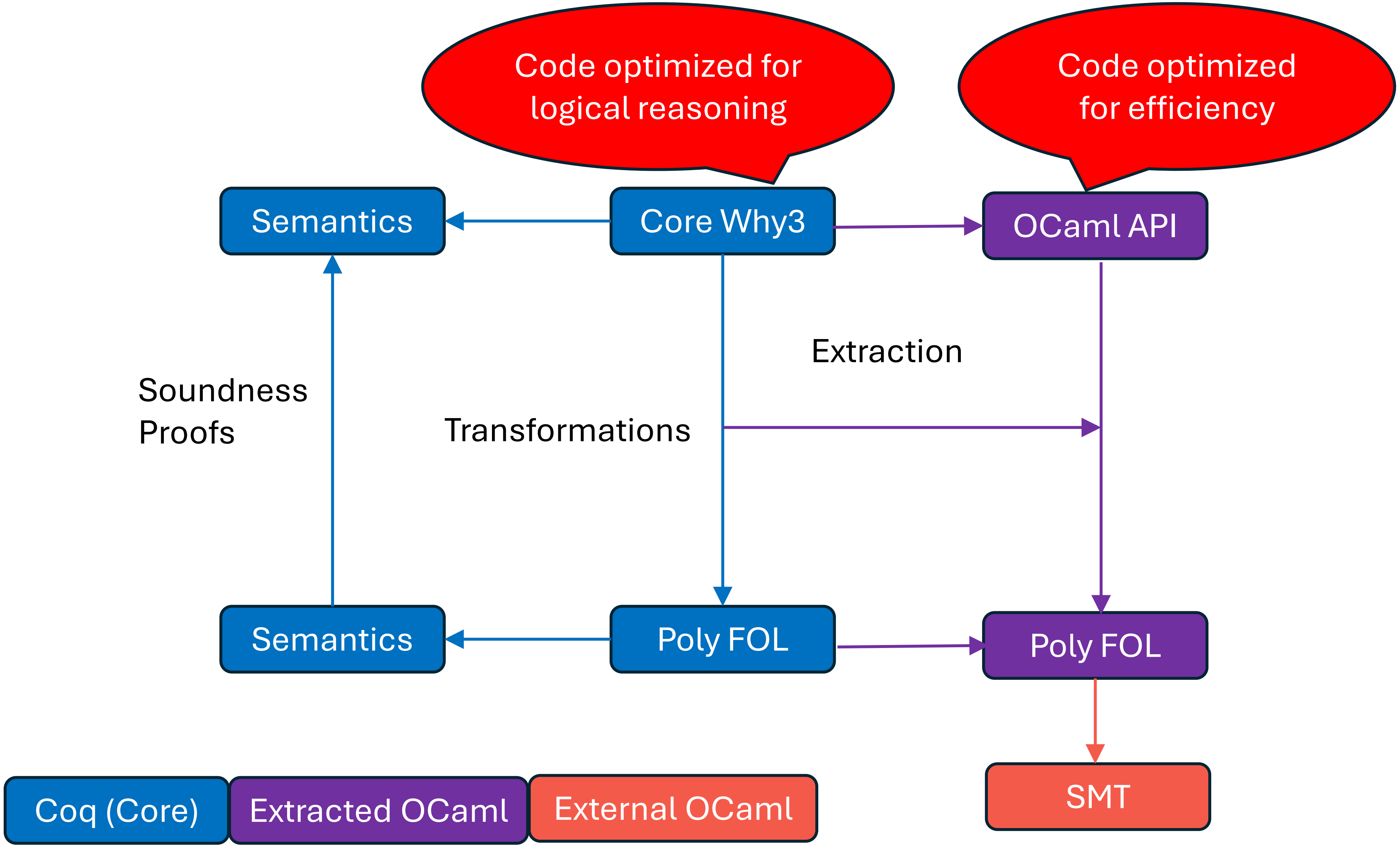

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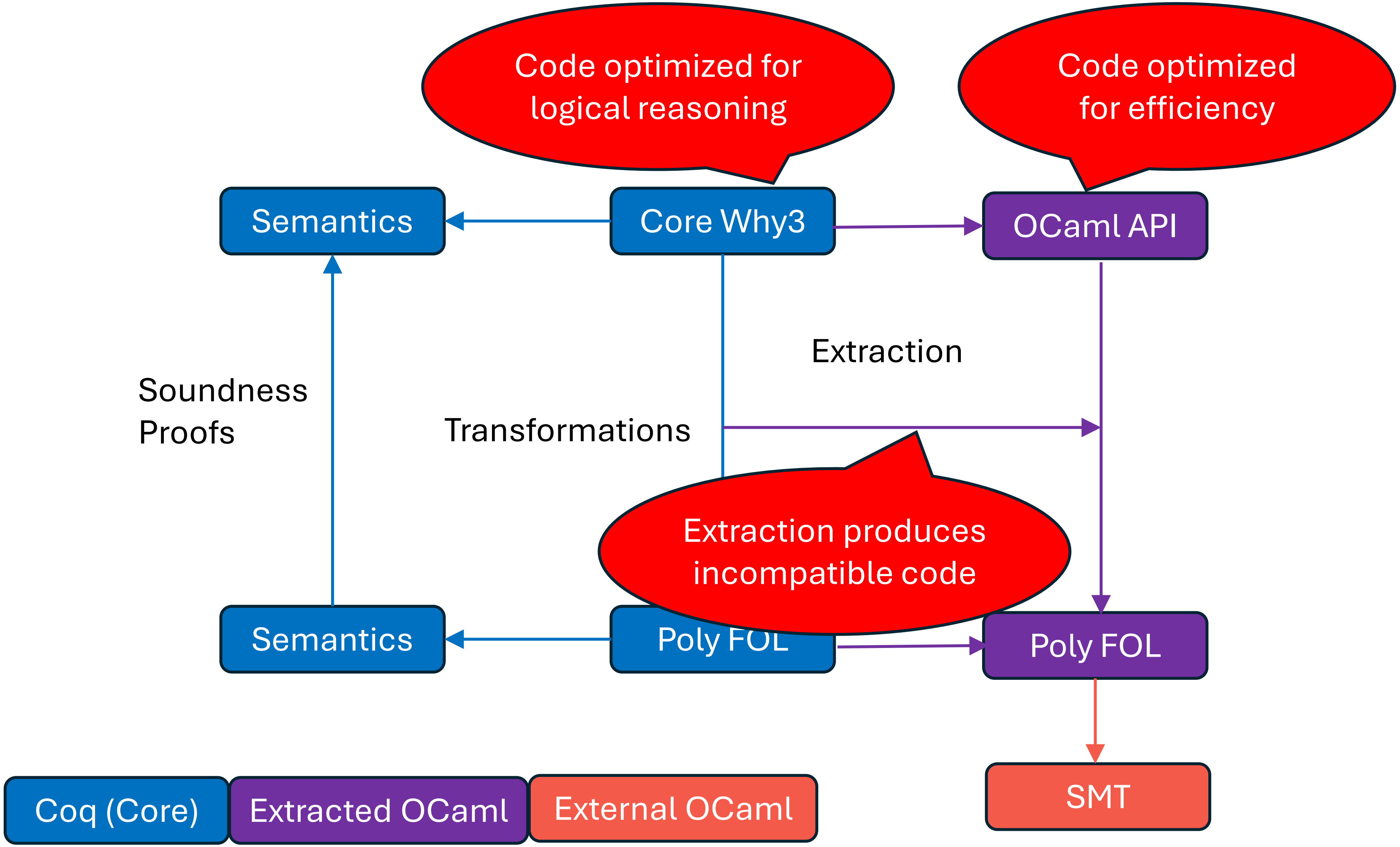

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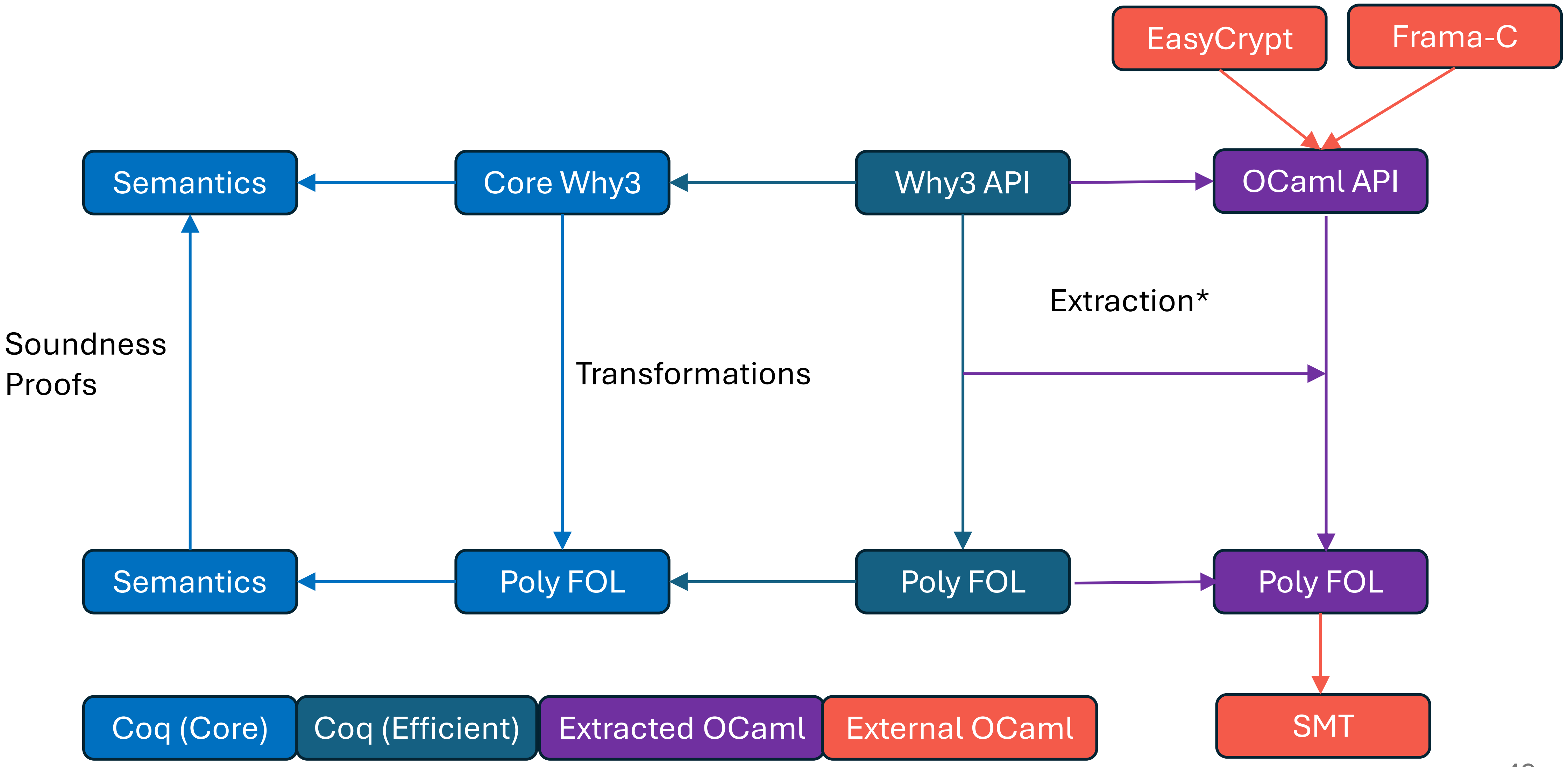

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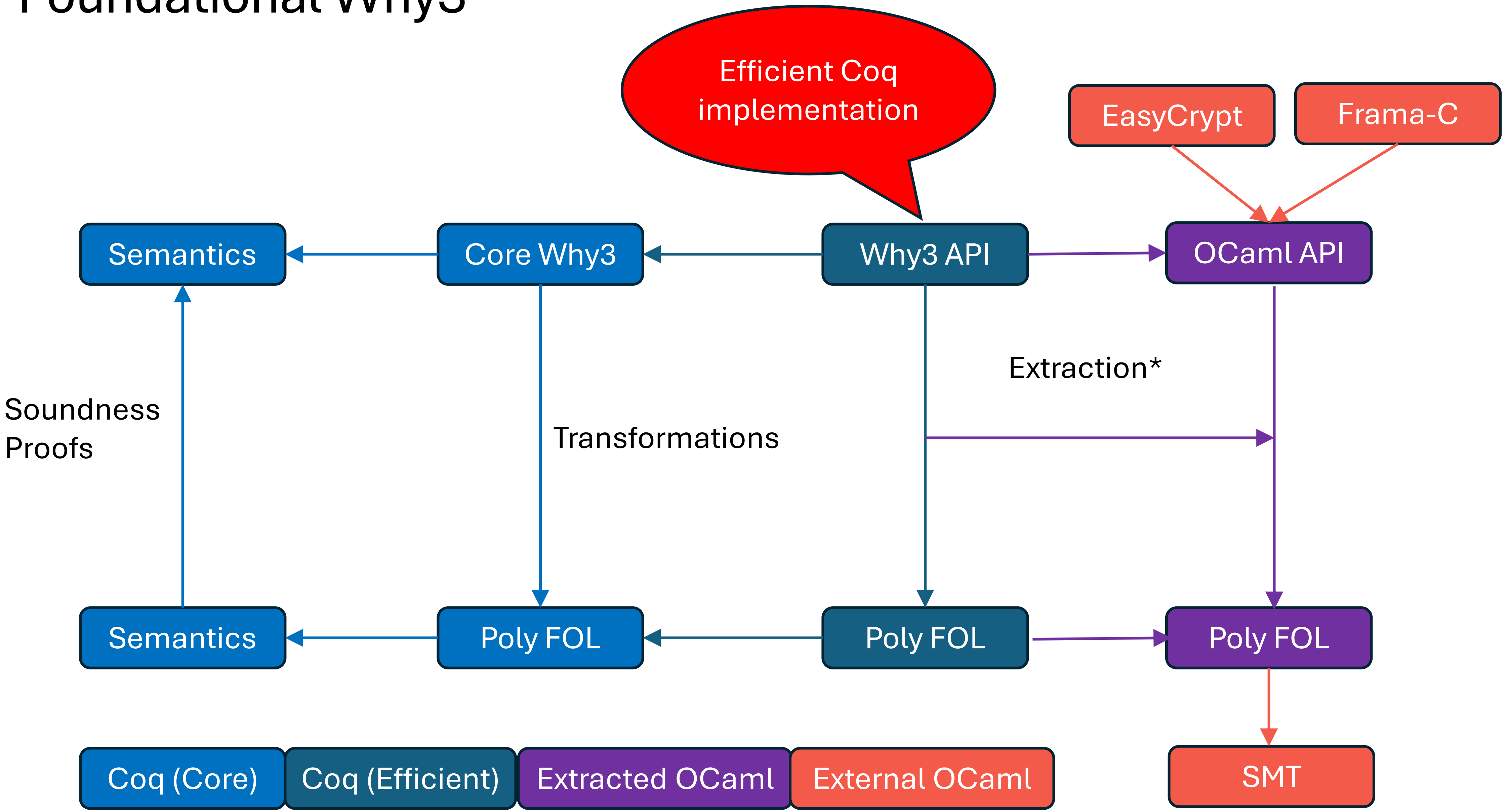

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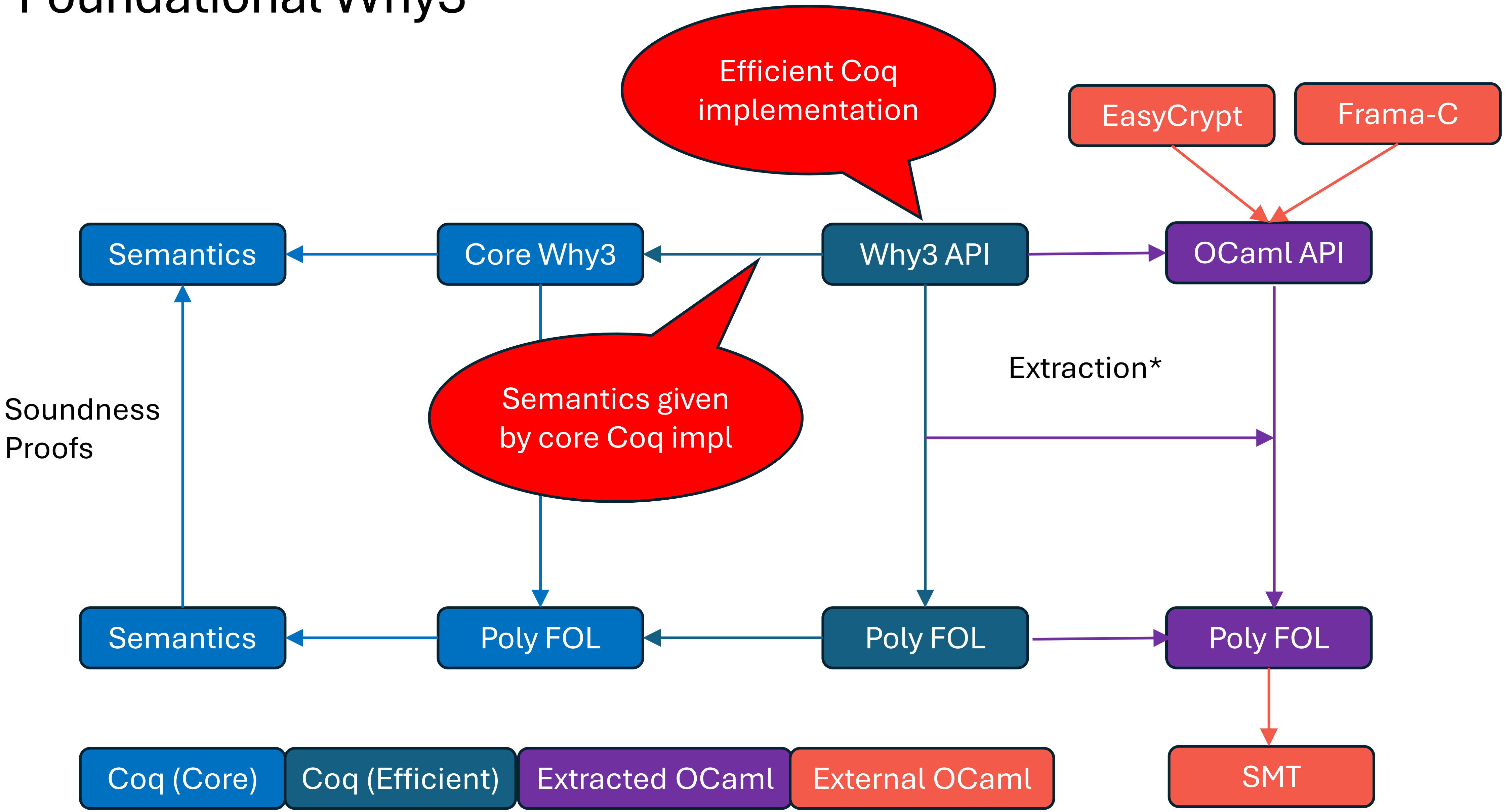

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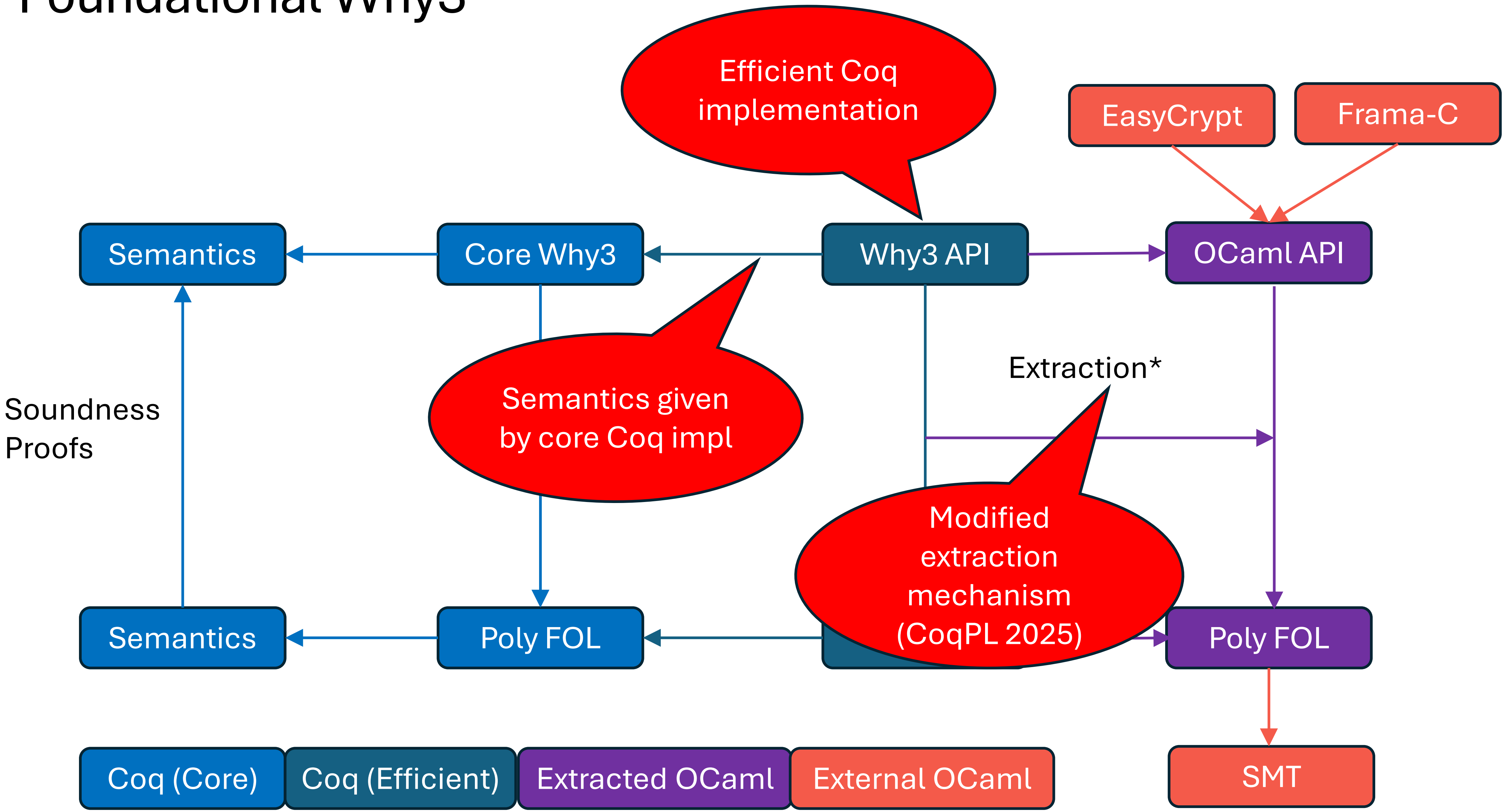

Implementing OCaml APIs in Coq - Example

```
val hd : 'a list -> 'a

let hd = function
    [] -> failwith "hd"
  | a::_ -> a
```

Similar ideas: [Nanevski et al. 2008] [Abrahamsson et al. 2020] [Sakaguchi 2020] [Boulmé 2021] [Korkut et al. 2025]

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```

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Modified extraction replaces monad with exceptions

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```

Modified extraction replaces monad with exceptions

Identical type and behavior as original!

```
let hd = function
| [] -> raise (Failure "hd")
| x :: _ -> x
```

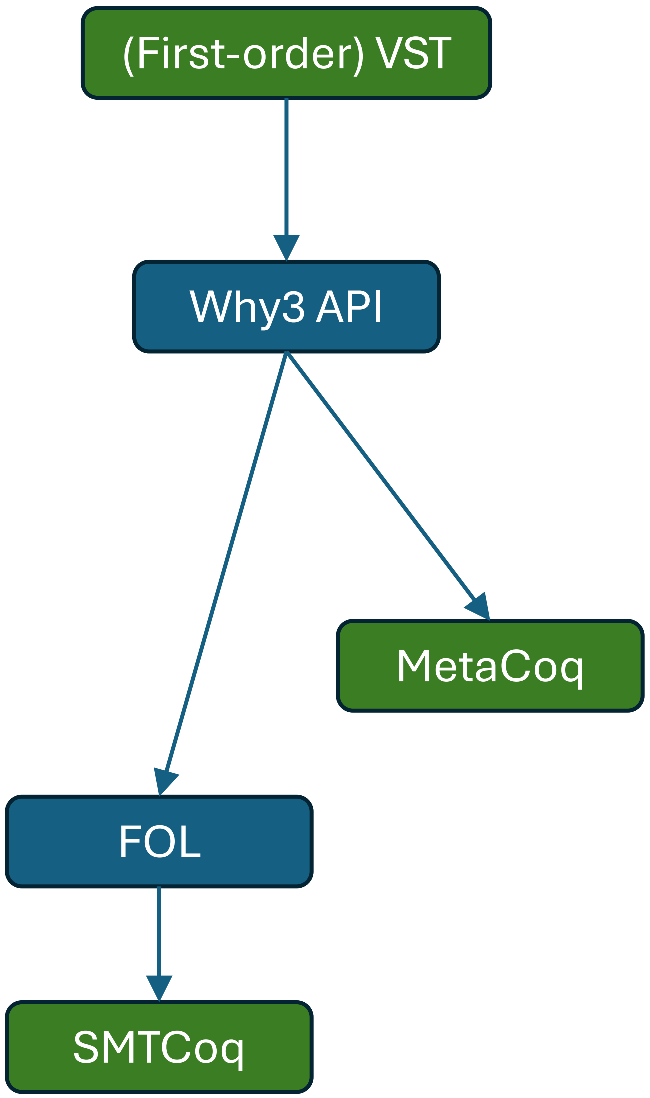
Similar ideas: [Nanevski et al. 2008] [Abrahamsson et al. 2020] [Sakaguchi 2020] [Boulmé 2021] [Korkut et al. 2025]

A New Why3 Implementation

- Handles large subset of Why3, not everything
  - Missing: lexicographic termination, function types, nested + nonuniform ADTs, interfacing with pure WhyML code
- Run Why3 and EasyCrypt test suites against Foundational Why3
- EasyCrypt good test: uses lots of features for real-world reasoning
- Pass all 183 EasyCrypt tests, on average 1.8x slowdown (~8 min vs 15 min), still practical!
- Main performance bottlenecks: functional hash tables, eager substitution, arbitrary-length integers
- Found several bugs in Why3

Conclusion

- Gave first:
  - Verified real-world IVL implementation
  - IVL implemented in a proof assistant
  - Formal semantics and proofs of soundness for recursive structures
- Future work:
  - Verify rest of (simpler) transformations for real-world use
  - Connect to other tools in front-end and back-end
- Implementation and proofs available at https://github.com/joscoh/why3-semantics
- Thanks for listening!



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