Implementing OCaml APIs in Coq

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    match l1 with
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    | x :: t => x :: (app t l2)
    end.
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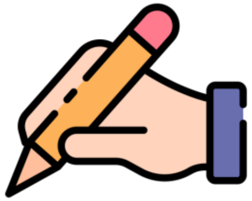

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(** val app : 'a1 list -> 'a1 list -> 'a1 list **)
let rec app l1 l2 = match l1 with
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module P1 = Kruskal.Make(G)(W)
let x = P1.spanningtree g
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foo.ml

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val append :
  'a list -> 'a list -> 'a list
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list.mli

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   let rec append l1 l2 = ...
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Software Analyzers

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Enables *incremental* verification

But, must match *exact* interface

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Software Analyzers

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  if ls.ls_constr <> 0 || ls.ls_proj then
    raise (UnexpectedProjOrConstr ls);
  let news = Sid.singleton ls.ls_name in
  mk_decl (Dparam ls) news
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Not reflected in type!

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Stateful (hash-consing)

OCaml Features Not (Idiomatically) Representable in Coq

- Exceptions
- Mutable State
- Opaque Types
- Reference Equality
- Machine-length integers(*)
- Mixed record-inductive types
- Etc

We want *computable* code in Coq and OCaml => No Axioms!

Our Solution

- A lightweight, pragmatic solution: represent features *differently in Coq and OCaml* by (carefully) modifying extraction

- We propose a *design principle* using this idea and provide a small library to enable programming with this pattern

Example: Exceptions

- Idea: implement error handling in Coq with error monad

Running example: hd

```
val hd : 'a list -> 'a

let hd = function
    [] -> failwith "hd"
  | a::_ -> a
```

Example: Exceptions

- Idea: implement error handling in Coq with error monad

Step 1: Model exceptions in Coq

```
Record errtype : Type := {errname : string; errargs : Type; errdata : errargs}.

Definition mk_errtype name {A} (x: A) :=
    {| errname := name; errargs := A; errdata := x |}.

Definition Failure (msg: string) : errtype := mk_errtype "Failure" msg.
```

Example: Exceptions

- Idea: implement error handling in Coq with error monad

Step 2: Define monadic interface (with coq-ext-lib)

```
Definition errorM A : Type := Datatypes.sum errtype A.

Definition err_ret {A} (x: A) : errorM A := ret x.

Definition err_bnd {A B} (f: A -> errorM B) (x: errorM A) : errorM B := bind x f.

Definition throw {A} (e: errtype) : errorM A := raise e.
```

Example: Exceptions

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Step 3: Erase monadic interface when extracting

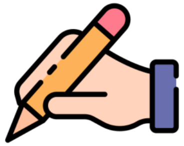
```
Extract Constant errorM "'a" => "'a".

Extract Inductive errtype => exn [""].

Extract Inlined Constant err_ret => "(fun x -> x)".

Extract Inlined Constant err_bnd => "(@@)".

Extract Inlined Constant Failure => "Failure".
```

Example: Exceptions

- Idea: implement error handling in Coq with error monad

Step 4: Implement API in Coq using interface

```
Definition hd {A: Type} (l: list A) : errorM A :=
    match l with
    | [] => throw (Failure "hd")
    | x :: _ => err_ret x
    end.
```



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Recipe

Given OCaml feature not representable in Coq:

- Identify abstract interface and model in Coq
- Implement primitives in Coq and map to OCaml
- Coq clients *only* use primitives to interact with feature

Result:

1. Coq code is computable, provable, and axiom-free
2. OCaml code is computable and has interface client expects (no monads)

Mutable State

- Implement state in Coq with state monad, in OCaml with mutable reference
  - `st A B` extracted to B
  - `bind` and `ret` same as error
  - Provide type for mutable references, extract to `'a ref`
  - `get` looks up in reference (!)
  - `set` sets value of reference (:=)
- Provide generic State module for creating state/reference of any type

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Solution: fix initial value!

```
Module Type State (T: ModTy).
Parameter create : unit -> st T.t unit.
Parameter get : unit -> st T.t T.t.
Parameter set : T.t -> st T.t unit.
Parameter runState : forall {A: Type}, st T.t A -> A.
End State.
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Run state starting from fixed value, OCaml resets mutable reference

Example: Integer Term API

```
type var
type tm_bound
type tm = private ... | Tvar of var | Tlet of tm * tm_bound

val create_var : string -> var
val t_open_bound: tm_bound -> (var * tm)
val t_close_bound: var -> tm -> tm_bound
val sub_t: var -> tm -> tm -> tm
```

Proving Things about State

- Variant of Hoare State Monad [TPHOLs '09] (shallow embedding)

```
Definition st_spec {A B: Type} (Pre: A -> Prop) (s: st A B)
  (Post: A -> B -> A -> Prop) : Prop :=
  forall i, Pre i -> Post i (fst (runState s i)) (snd (runState s i)).
```

- Use this to prove substitution correct

```
Theorem sub_t_rep (tm1: tm):
  st_spec (fun i => tm_st_wf i t /\ tm_st_wf i tm1 /\ var_st_wf i v)
    (sub_t v t tm1)
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If input well-formed (variables <= current state)

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After running stateful function sub_t

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The result coincides with semantic substitution

Limitations

- Difficult to scale, need e.g. new definition/extraction directive for each exception
- Need dune preprocessing, not difficult but repetitive
- Need to redefine monadic operations for each monad to avoid Object.magic in OCaml code
  - Could be solved with a hypothetical `Extraction cbv foo` command
- Enlarge TCB with extraction directives
- Big limitation: cannot enforce that Coq client only uses interface!

Related + Future Work

- Ynot [ICFP ’08] - framework for imperative programming, axiomatizes stateful operations
- ITrees [POPL ‘20] – model impure code with coinductive monads, cannot compute in Coq
- Proof-certificate-producing stateful CakeML from monadic HOL [IJCAR ‘18]
    - Could be used to certify correctness (with an OCaml program logic)
- FVDP [Boulmé Thesis ‘21] – use may-return monad for untrusted OCaml oracle
    - Could allow generated OCaml APIs to depend on other OCaml code
- VeriFFI [POPL ‘25] – verified FFI between Coq and C
    - Solves opacity issue by axiomatizing foreign functions, giving rewrite rules

Conclusion

- Our approach aims to be *lightweight* and *practical*
- Resulting programs:
  - Executable in both OCaml and Coq
  - Usable by clients in both languages
  - Can be reasoned about in Coq using ordinary Coq logic
- Code available at https://github.com/joscoh/coq-ocaml-api
- Includes examples: List API, mutable counter, term API
- Thanks for listening!

References

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